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Supplement to Final Report

1 June 1966 Through 30 May 1970

Prepared for
Geophysics Division
Air Force Office of Scientific Research
Arlington, Virginia 22209

By
P. L. JACKSON

August 1970

GEOPHYSICS LABORATORY
Willow Run Laboratories
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Sponsored by
Advanced Research Projects Agency
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THE UNIVERSITY OF MICHIGAN
Ann Arbor, Michigan

FOREWORD

The research described in this report was conducted by the Geophysics Laboratory of Willow Run Laboratories, a unit of The University of Michigan's Institute of Science and Technology. The work was performed as part of Project VELA UNIFORM, sponsored by the Advanced Research Projects Agency and monitored by the Air Force Office of Scientific Research under Contract No. AF 49(638)-1759. The research period extended from 1 June 1966 through 30 May 1970; the Project Scientist is Mr. William J. Best.

The principal investigators for this project were P. Jackson, R. Turpening, and D. Willis. This report was submitted for publication in July 1970. The Willow Run Laboratories' report number is 8071-33-F_1.

PREFACE

The investigation reported in this dissertation was conducted at the Geophysics Laboratory of the Institute of Science and Technology, The University of Michigan, over a three-year period. During this period the preliminary results of this work have been described in presentations, reports, and a journal article.

On April 5, 1968, a slide presentation of computer-drawn plots was made to the Geophysical Advisory Committee of the U. S. Air Force Office of Scientific Research, Alexandria, Virginia.

On April 13, 1968, a paper concerning this work was presented to the Annual Meeting of the Seismological Society of America, in Tucson, Arizona (Jackson, 1968).

A major report, including listings and flow diagrams of computer programs was submitted in September, 1968 to the Air Force Office of Scientific Research (Willis and Jackson, 1968).

On October 5, 1969, a paper was presented to the Annual Meeting of the Eastern Section of the Seismological Society of America in Blacksburg, Virginia (Jackson, 1969).

A journal article describing the application to a spherical earth appeared in the Bulletin of the Seismological Society of America in June, 1970 (Jackson, 1970).

During this three-year period numerous progress reports on this investigation have been submitted to the Air Force Office of Scientific Research and the American Petroleum Institute.

The author would like to thank David E. Willis, James T. Wilson, Charles G. Bufe, Henry N. Pollack, Paul W. Pomeroy, Timothy C. Swanson, and Roger M. Turpening for helpful suggestions and encouragement. He also would like to thank Mrs. Clara M. Randazzo for invaluable help in preparing the manuscript.

ABSTRACT

Simulation of seismic rays for a spherical earth and a flat earth has been achieved in highly complex models. Travel times and approximate amplitudes of seismic waves can be found for both two- and three-dimensional models of portions of the earth. In seismology and other disciplines ray construction customarily has been applied to simplified geometries. It has been necessary to assume that the seismic wave velocity distribution of the earth was relatively uniform and symmetric.

Recently, however, the earth has been found to be more complex and non-uniform than formerly assumed. A need has thus arisen in seismology to test highly heterogeneous models of seismic velocity distribution. At the same time the development of the modern digital computer has provided a means of performing the necessary ray constructions and numerical calculations.

The problem of complicated seismic velocity distributions was therefore investigated in terms of the most appropriate use of the digital computer. For this investigation a velocity field was set up, and the propagation computations made for short segments of rays within this field. Total travel times are found by adding the travel times of connected ray segments. Essentially, the nature of propagation was duplicated on the computer, in that, at the location of each segment along the path of propagation, the initial condition and effect of the surroundings determine the succeeding direction of the following segment.

Both visual and numerical results have shown that this simulation method can be usefully applied to investigation of seismic velocity distributions of portions of the earth of any size or complexity.

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Introduction

Possibly no artificial construction has been more useful to science than the ancient concept of the ray. Not only in seismology, but in all scientific fields concerned with the transfer of energy, reasoning with and construction of rays have resulted in fundamental advances.

The firm basis of rays in science is most dramatically shown in optics. From ancient times to the present, light rays have been used for intuitive visualization and have resulted in advances in optics and related fields. Archimedes, Aristotle, Roger Bacon, Kepler, Newton, Young, Fresnel, Rutherford, and Compton all used the ray concept. Newton's Optiks (1730; 1952) is probably the most lucid example of rays as both a rationale and a means of intuitive understanding and imaginative discovery; almost every figure in Newton's book is a ray tracing.

In seismology, with which this thesis is concerned, the use of rays aided Mohovoričić (1910) in discovering the discontinuity between the mantle and the crust; Gutenberg (1914) inferred the existence and estimated the size of the earth's core with the aid of rays; and, in theoretical analysis, Zoeppritz (1919) employed the ray concept to derive the relationships of reflected and refracted dilatational and distortional waves at an interface.

Current seismological contributions to the knowledge of the earth continue to rely on the concept of rays. Current issues of scientific journals in seismology usually contain several articles which are illustrated by and rely on seismic rays.

In seismics most ray tracing has been based on Herglotz-Wiechert formulation as extended by Bullen (1963). Helbig (1965) used this geometrical construction for a graphical method for spherical shells. Spherically symmetric ray tracing has also been described by Julian and Anderson (1968) and shown by Lewis and Meyer (1968). Engdahl, et al. (1968) employed the formulation for the 1968 P-Phase Tables (Taggart, et al., 1968).

Iyer and Punton (1963) broke away from rigid geometrical limitations in constructing successive wavefronts by applying Huygens' principle involving complicated logic. Yacoub, et al. (1968) also departed from rigid geometry by considering regions of constant velocity in which the interfaces can be oriented in any direction. He also computed the amplitudes by Zoeppritz's equations for rays crossing an interface.

The ray simulation described in this dissertation is based upon a new concept leading to the treatment of heterogeneous structures and the travel-time solutions for any discretely specified and/or analytically represented velocity distributions.

An advance in the art of tracing rays is thus a significant contribution to the science of seismology; and, by extension, to other sciences. Currently two aspects of recent scientific and technological development provide, first, a new requirement and, second, a practical means of satisfying this requirement.

The requirement to investigate non-simplified, heterogeneous velocity distributions, has arisen from the discovery that the earth is less symmetric and more heterogeneous than previously supposed. More accurate

and extensive seismic measurements are currently being made. The need for greater understanding has made simplifying assumptions of the velocity distribution of the earth less useful. Tracing rays through heterogeneous velocity structures and determining their travel times would be useful in gaining detailed understanding of the earth.

The practical means to achieve the tracing of rays and determination of travel times through complex velocity distributions is the employment of the modern digital computer. Complicated velocity distributions imply unpredictable velocity gradients. In large regions a ray must be constructed with many different computations because of changes within relatively small regions. With a digital computer one can carry out successive computations, which enables one to treat the complicated velocity distributions indicated by modern seismology.

With these two aspects in mind—the requirement and the apparent means of fulfilling the requirement—a preliminary trial was made to develop a new method of ray tracing. The point of departure was to somehow propagate rays through some type of mathematical or representational cross-section of a velocity distribution. The approach was to use a sampled structure into which a short segment of a ray could be introduced at a given location and with which the directions and positions of successively added segments could be found as functions of the velocity distribution represented by the structure.

A rectangular two-dimensional grid of equally-spaced points was considered. The grid points represented positions on a vertical cross-section of the earth, each point corresponding to a given horizontal and

vertical distance from a reference point. A "sampled" velocity and slope would correspond to each point of the grid. A means was then required of using the velocities and slopes, and the relationships between neighboring samples of their values, to "propagate" short successive segments of the rays through the sampled velocity distribution. This means was found. An introductory description follows.

The aforementioned grid was considered as a matrix in which the rows represented discrete horizontal distances along the cross-section, while the columns represented discrete vertical distances, as shown in Figure 1. Such a matrix can be represented as a vector with two indices. Call the vector V_{ij}, in which i represents the row, and j the column of the matrix. For each set of indices (i,j) a position on the cross-section can be identified. For example, let the interval between the samples be 10 km. Then the indices (i = 1, j = 1) could represent the reference point (0,0) at the surface of the earth and on the cross-section; the indices (i = 10, j = 1) represent a distance 90 km from the reference point on the surface; the indices (i = 5, j = 3) a horizontal distance 40 km from the reference point at a depth of 20 km. The indices must be integers greater than zero to represent the matrix in the computer. For this reason the smallest pair of indices (1,1) are taken to represent an origin (0,0). This bias is easily handled, as the source can be anywhere within the bounds of the matrix, provision for measuring from the source location must be made.

At each location defined by two indices there corresponds a velocity V_{ij} and a slope S_{ij}. These values at discrete points can be referenced

from locations closest to them. A ray, however, is a succession of short segments of possibly differing directions. These segments may be joined together at non-discrete locations, say the coordinates (h,d), where h is horizontal distance and d is depth in a cross-section of the earth. At each matrix location (h,d) a computation must be made to determine the direction and hence end position of a succeeding segment. The ray segment midpoints could then be referred to the nearest discrete (i,j) location to determine the velocity V and slope S. For example, consider the scale, or sampling interval as shown in Figure 1 was 10 km, and the ray segment midpoint was located with respect to the matrix at h = 4.75, d = 3.25 corresponding to H = 37.5 km and D = 22.5 km on the referenced cross-section of the earth. The velocity V and slope S would be found at the location indicated by the indices (i = 5, j = 3), and could be easily referenced; if desired, one could interpolate the values between those found at (i = 5, j = 3) and (i = 4, j = 4).

At the midpoint of every ray segment a value for velocity and slope is found. For geometrical ray tracing, Snell's law requires the knowledge of an incident direction with respect to an interface and the velocity of the medium on either side of the interface. The representational structure described above supplies this information. The direction of the incident ray segment and the slope of the interface are known, therefore the incident angle for Snell's law can be found. The velocity V_1 of the medium on the incident side of the interface is found as described in the last paragraph. The velocity V_2 on the emerging side can be found by extending the ray segment in the same direction as the incident ray for

the segment, again using the technique described in the last paragraph.

For a fixed rectangular Cartesian coordinate system, a horizontal distance can be represented as $L \sin \theta$ and a vertical distance as $L \cos \theta$, where L is a length of a ray segment and θ is the angle from the upward vertical. θ is positive in the counterclockwise direction. Now consider that the head of an incident ray segment of angle θ_k located at (h_k, d_k), and, by invoking Snell's law the emerging angle is found to be θ_m, as shown in Figure 1. The location of the tail end of the emerging ray segment is found to be $h_e = h_k + 0.5 L \sin \theta$, $d_e = d_k + 0.5 L \cos \theta$. As the direction and location of the emerging ray segment as well as the velocities and slopes of the immediate surrounding regions are known whenever they may be within the defined matrix, the emerging ray segment can be redefined as the new incident ray segment, and succeeding segments generated and joined end-to-end until a complete ray is formed.

Travel times of the radiation of seismic waves from the disturbance to the seismometer are of fundamental importance in seismology. The information to determine travel times is available with this ray tracing method. Each segment of the ray is located in a region in which the velocity is known. The travel time for each segment can be found by dividing the length of the segment by the average velocity along its length. The travel time for the ray is then the sum of the travel times of the segments. The distance of travel is found by summing the lengths of the segments. This distance is useful for computing spherical spreading and logarithmic attenuation.

The approach described above was tested with a computer program. The initial results were promising, and the ideas seemed to be valid. A seismic ray could be simulated through a matrix representing a sampled velocity distribution in a vertical cross-section of the earth. The investigation to produce a useful, accurate, and versatile seismic ray simulation was then commenced. The ensuing investigation was mainly devoted to attacking the following problems:

1. Accommodating ray angles through the entire range of 360°, and of using any defined slope in association with any given ray angle,

2. Reflection from an interface,

3. Both critical and non-critical multiple reflections,

4. Precisely locating an interface between matrix-designated discrete locations and extending the ray to this interface,

5. Developing and testing a method to obtain high accuracy, of determining travel times and approximate amplitude in a multiply-reflecting model,

6. Adding the capability of specifying regions of velocity characteristics defined by an analytical mathematical function,

7. Treating both curved and flat earth,

8. Applying the concept to a three-dimensional model, and

9. Making the method general so that one program can be used for many different regions of the earth.

A versatile tool has been developed for the scientific investigation of velocity distribution in the earth. Any conceivable velocity distribution can be simulated and compared to actual travel times in

investigations where geometrical ray approximations are valid. In addition, the method is expandable, and can be used as a base for mode conversion, the addition of Zoeppritz's equations for amplitude, extension to the atmosphere where winds provide a moving medium, and, possibly, for geometric diffraction.

The simulation of seismic rays can be performed with either two- or three-dimensional models. Flat earth models naturally accommodate themselves to a rectangular Cartesian coordinate system. Curved earth models, which at first glance would appear to be better treated in polar coordinates, are also treated in a rectangular Cartesian system because the velocities and slopes for a spherical earth are easily referenced. Also the compatibility of coordinate systems between flat and spherical earth enables one to insert sampled values for anomalous regions within a spherical earth model.

As first conceived, it was thought that the two-dimensional technique of ray tracing could be incorporated into the three-dimensional technique, and two-dimensional tracing performed when holding the values along one of the axes of the three dimensions constant. However, the use of three-dimensional models required a sufficiently different technique that the two types are better explained separately. The description of the use of the three-dimensional model is not self-contained. To avoid repetition of identical material, pertinent matter presented in the two-dimensional section will be referred to when describing its use in three dimensions.

Two-Dimensional Simulation

Flat Earth

Both discrete values assigned to a matrix and continuous mathematical functions may be employed to reference velocity. To aid in visualization, the following exposition is based on the discrete case.

Construct a two-dimensional scalar field representing the seismic velocity characteristics and slopes along a plane in a medium, such that

$$V = f(x,z) \quad (1)$$

$$S = g(x,z) = h(V) \quad (2)$$

where V is the velocity (actually the speed with which seismic P-waves propagate, but to be referred to henceforth as velocity, in keeping with common practice in seismology), f, g, and h are functions, and x,z represent distances parallel to the axes of a rectangular Cartesian coordinate system. The functions f and g may be defined differently for specified regions of the field, and can include both continuous mathematical functions and references to tables of arbitrary and discretely changing values. The function g may be derived directly from f as the normal to the gradient, or specified separately. One might term S a vector, as it corresponds to a direction and is based on the gradient. However, it is defined as an angle and is used as a scalar quantity in the computation.

Consider a matrix of discrete values, the rows of which represent equally spaced horizontal positions within the scalar field. For each element of the matrix a velocity and slope is assigned which corresponds to the location represented in the scalar field. Values for velocities and slopes can be found in the scalar field at any point not corresponding

to the discrete locations represented by the matrix elements. These values are found by either using those of the matrix elements to which the location is closest or interpolating between the nearest horizontal and vertical elements of the matrix. Thus for any position in the scalar field, the value for the velocity and slope can be found by reference to the nearest element of the matrix.

As arbitrary values can be stored in the matrix, any type of velocity distribution can be referenced to the scalar field. Through such a reference system a completely heterogeneous velocity distribution can be used for ray tracing; the limits of heterogeneity are limited only by the detail with which the matrix elements represent the scalar field. That is, by the distance represented by two adjacent elements of the matrix. The matrix with the velocities and slopes represented as its elements is necessary to arbitrarily represent velocity distributions, but not to perform ray tracing through the scalar field.

Select a position x_k, z_k for the head end of an incident ray segment of length L and an angle $\theta_k (0° < \theta < 360°)$. The midpoint of this segment is the location which is used to determine the incident velocity V_k in the manner described above. Reference θ_k to the negative z-axis, counterclockwise positive. The z-axis, which represents depth in the earth cross-section, is positive downward. Extend the ray segment a distance L in the direction θ_k.

The initial emerging location P_1 is found by extending the ray segment at the incident angle θ_k, shown in Figure 1, using the two following equations:

$$x_p = x_k + L \sin \theta_k \tag{3a}$$

$$z_p = z_k + L \cos \theta_k \tag{3b}$$

The midpoint of this extended ray segment is the location which is used to determine the initial emerging velocity V_m.

Sufficient information (position and angle of the ray, and the velocities and slopes surrounding the region of the ray) is available to construct the ray continuation by Snell's law. The equation used for this purpose is

$$\theta_m = \text{ARCSIN}[(V_m/V_k) \sin (\theta_k - S_k)] + S_k \tag{4}$$

where θ_m is the angle of emergence, V_m is the emerging velocity, V_k is the incident velocity, and $S_k (-90° < S \leq 90°)$ is the slope of the interface referenced counterclockwise positive from the x-axis. In case of continuous velocity functions S_k is the normal to the direction of the velocity gradient. The extension of this segment is termed "probing" in the sense that one must repetitively probe for the proper emerging angle θ_m. For the first repetition θ_m replaces θ_k in Equation (3) and P_x, P_y are recomputed. A new θ_m is determined using the newly-found V_m in Equation (4). Equations (3) and (4) are repeated in succession, each time using the previously determined value of θ_m for θ_k in Equation (3). The repetition is terminated when successive values of θ_m are sufficiently close in value ($|\theta_m - \theta_{m-1}| < \varepsilon$, where ε is a predefined small value).

This repetition is necessary because the location of the extension computed with the incident angle is in general different from the location computed with the emerging angle. If the ray is penetrating into

a region of increasing velocity the emerging angle between the ray segment and the normal to the interface will be too large. If penetrating into a region of decreasing velocity this angle will be too small. These improperly evaluated angles are found because the tentative ratio V_m/V_k may be a different value from the ratio as found when extending the ray with the correct emerging angle. One can asymptotically converge to the correct emerging angle by repeating these computations. These repetitions are performed to improve accuracy for computations using any given ray segment length L. Linearity of the velocity function is assumed within the distance L. Also, to improve accuracy one might reduce the segment length L, as the accuracy is an inverse function of L, and L is not required to be equal length for all ray segments. However, it still must hold that the difference between two successively-determined emerging angles θ_m would have to be a small value.

When the sufficiently accurate θ_m is found by repeatedly applying Equations (3) and (4), the emerging ray segment and angle is determined. This emerging segment and angle is then treated as the incident segment with a new location and angle. Segments are repeatedly computed, the tail of the "emerging" segment joined to the head of the "incident" segment repeatedly until the ray, which is the summation of all the segments, penetrates to the surface or the boundary of the defined cross-section of the earth.

Spherical Earth

Consider a velocity distribution with depth for a symmetric earth.

One can compute the velocity and slope for any position (x,y) within a circle representing the cross-section of the entire earth. The velocity is found as follows:

$$V_k = f(R) = f[(x_k{}^2 + y_k{}^2)^{\frac{1}{2}}] \tag{5}$$

where f is a function of the radius R; V_k is found for any location defined by (x_k, y_k) from either a table and interpolating, or by computing some mathematical function of the radius $R = (x^2 + y^2)^{\frac{1}{2}}$.

For each location defined by (x_k, y_k) a slope S_k can be found,

$$S_k = \text{ARCTAN } (y_k/x_k) \pm \pi/2 \tag{6}$$

where y and x are the coordinates of a given location, y=0, x=0 are the coordinates of the center of the earth, and the sign of $\pi/2$ depenas upon the quadrant in which y and x are found.

The angular distance at which a ray emerges is found by

$$\Delta = \text{ARCTAN } (y_k/x_k) \tag{7}$$

where the proper quadrant is determined from the relationships of the signs of x and y, and where x and y are located within a small predefined distance from the surface of the earth.

The depth of penetration, or, correspondingly, the minimum radius of the ray upon reflection is found by computing the minimum radius along a given ray, and retaining that radius to correspond with the other previously described data of the emerging ray.

Any region which is defined by minimum and maximum values or functions of x, y, or R can be postulated so that a different function can be invoked. For example, one might wish to investigate the travel times of rays which penetrate an anomalous region in an otherwise symmetric

region of the earth. Within the limits defined, any sampled value can be inserted for the positions defined by x and y, as in the flat earth case.

Three-Dimensional Simulation

Construct a three-dimensional scalar field representing the seismic velocity characteristics of a medium, such that

$$V = f(x,y,z) \tag{8}$$

$$S_1 = g(x,y,z) \tag{9}$$

$$S_2 = h(x,y,z) \tag{10}$$

where, as in the two-dimensional fields V is the velocity, S_1 represents the slope with respect to the z-axis, S_2 the slope along planes parallel to the x,y plane, and f, g, and h are functions. Positions within the field are referenced in a three-dimensional orthogonal Cartesian coordinate system. Directions within the field are referenced in a spherical coordinate system, in which the angle from the positive z-axis is $\phi(0<\phi<\pi)$ and the direction from the z-axis in a plane parallel to the x,y plane is the angle $\theta(0<\theta<2\pi)$, the same as that described for two dimensions. A segment L oriented in a direction defined by ϕ and θ projects a distance P_z on the z-axis

$$P_z = L \cos \phi \tag{11}$$

and on the x-axis as

$$P_x = L \sin \phi \sin \theta, \tag{12}$$

and on the y-axis as

$$P_y = L \sin \phi \cos \theta. \tag{13}$$

To visualize the role of the two slopes $S_1(-\pi/2<S_1<\pi/2)$ and $S_2(0<S_2<2\pi)$ more clearly, consider an interface or a plane normal to the gradient which lies obliquely to all of the axes. Then the slope S_1 is the dihedral angle between the z-axis and the plane of the interface (when interface is mentioned it also refers to the normal plane to the gradient). The slope S_2 is found by the angle of the line which defines the intersection between the x,y plane and the plane of the interface, and is referenced as in the two-dimensional case.

The coordinate system is a right-handed system in which the z-axis is downward. Thus when a region of the earth is represented positive z values correspond to depth from the surface of the earth, or from an arbitrary horizontal boundary. Although an orthogonal Cartesian coordinate system is employed for position, a new position can be determined by employing a length L, and the angles ϕ and θ. The two coordinate systems are complementary.

We see from the foregoing definitions that, as in the case of the two-dimensional system, we have sufficient information to compute Snell's law in the three-dimensional system: L, ϕ, θ, x, y, z, V, S_1, and S_2. However, in the three-dimensional system the algebraic and trigonometric processes become much more involved.

Consider a velocity distribution in which the gradient is everywhere parallel to the z-axis as defined above. Such a distribution would correspond to a region of the earth in which all interfaces were parallel to the surface of the earth. Therefore, no velocity change would occur along a horizontal direction. With such a velocity distribution, the

original angle θ_k of a ray would never change, although its sign might. This is because $V_m = V_k$ and, therefore $\theta_m = \theta_k$, as seen by reference to Equation (4). The straightness of the ray is illustrated in the orthographic projection in the x,y plane of Figure 8. The total change in direction will be in the angle ϕ, which can be computed as

$$\phi_m = \mathrm{ARCSIN}[(V_m/V_k)\ \sin(\phi_k - S_{1k})] + S_{1k} \tag{14}$$

where the subscripts m and k refer to emergence and incidence as in the two-dimensional case.

We now consider a scalar field in which the interfaces may be any orientation in three-dimensional space; the condition described in the last paragraph would not hold for this case. If the coordinates were oriented in such a manner that the z-axis was perpendicular to the interface, then the condition holds where θ does not need to be recomputed and a simple expression for ϕ, Equation (14), can be employed. At any given location within the field the angles θ_k and ϕ_k are known, as well as the slopes S_{1k} and S_{2k}. If a coordinate system can be found in which the z-axis is perpendicular to the interface defined by S_{1k} and S_{2k}, then the calculation of Snell's law for ϕ_m in Equation (14) needs only to be used.

Such a coordinate system can be found by rotating the axes as a function of S_{1k} and S_{2k}. After rotation the positions x,y,z are referenced by x',y',z' and the angles ϕ and θ by ϕ' and θ' to the new orientations of the coordinate system axes. Snell's law can then be invoked with parameters referenced to the new coordinate system. The new position is found as follows.

Let S_2' be the angle by which the x- and y-axes are rotated around the z-axis, and S_1' the angle by which the x'-axis is rotated about the y'-axis (or, viewed differently, the angle with which the z-axis is tilted to become normal to the interface.) For a continuous mathematical function angle S_2' is

$$S_2' = (\pi/2) - \arctan(\Delta X/\Delta Y) \tag{15}$$

where ΔX and ΔY are directional derivatives along the x-axis and y-axis respectively. S_1' is found similarly as

$$S_1' = (\pi/2) - \arccos(\Delta Z/\Delta X^2 + \Delta Y^2 + \Delta Z^2)^{\frac{1}{2}} \tag{16}$$

where ΔZ is the directional derivative along the z-axis. For sampled functions S_1' and S_2' are

$$S_1' = (\pi/2) - S_1(i,j,k) \tag{17}$$

and

$$S_2' = S_2(i,j,k) \tag{18}$$

respectively.

The coordinate axes can then be rotated, and corresponding locations in the new coordinate system found by rotating first around the z-axis and next around the y'-axis:

$$X'' = X \cos(S'_2) + Y \sin(S_2') \tag{19}$$

$$Y' = Y \cos(S_2') - X \sin(S_2') \tag{20}$$

$$X' = X'' \cos(S_1') - Z \sin(S_1') \tag{21}$$

$$Z' = Z \cos(S_1') + X'' \sin(S_1') \tag{22}$$

The angles θ' and ϕ' are found in a similar manner by computing a position with trigonometric functions, rotating the coordinate axes, and taking inverse trigonometric functions of the angles of a directed vector from the origin to the position in the new coordinate system.

When each segment of the ray is computed, the new position can be found by rotating the coordinates back into the originally specified coordinate system. From this position in the given coordinate system the incident velocity V_k is known, and by repeated computation in the manner described in the section on two-dimensional tracing the proper emerging angles θ_m and ϕ_m can be asymptotically approached to within a predetermined small value.

In returning the rotated coordinate system, the values V'_k and V'_m required to compute Snell's law to determine the emerging angle ϕ'_m, are

$$V'_k = V_k \tag{23}$$

$$V'_m = V_k + (V_m - V_k) \cos (\phi'_m) \tag{24}$$

where, again, the prime refers to values in the new coordinate system, and ϕ'_m refers to the repetitively probed value to correctly determine V_m.

In the rotated coordinate system we are concerned with only one direction of reflection, so this can be handled in the same manner as in Equation (30) of the next section. The only angle affected is ϕ', as θ' will be unaffected. However, in the originally given coordinate system, the direction of the ray may reverse itself in its projection on the x,y plane.

Auxiliery Computations

Travel Times

As each segment of the ray is found and added on to the preceding segments, the travel time can be determined. The time of trevel for each segment is simply the length of the individual segment L_e divided

by the average velocity V_a at which it is propagating, so that the total travel time TT of a ray is

$$TT = \sum_{a=1}^{n} L_a/V_a \tag{25}$$

where n is the total number of segments from the ray's origin to its emergence on the surface.

Approximation of Amplitude

The total distance D a ray has travelled is

$$D = \sum_{a=1}^{n} L_a \tag{26}$$

Knowing this distance, one can compute the amplitude dimension due to spherical spreading and the exponential attenuation to energy dissipation as

$$A_e = A_s(1/D)\exp(-cD) \tag{27}$$

where A_e is the amplitude at the receiver, A_s is the amplitude at the source, and c is the attenuation coefficient.

In case of non-critically reflected rays and the ensuing refracted rays, the partition of energy is rigorously computed by Zoeppritz's equations. The approximative approach used here is that of Fresnel's reflection coefficient at normal incidence. At a reflective interface where non-critical reflection occurs the approximate reflected amplitude A_r is

$$A_r = A_k(V_k - V_{k+1})/(V_k + V_{k+1}) \tag{28}$$

and the subsequently refracted amplitude is

$$A_{k+1} = A_k - A_r \tag{29}$$

Reflection

Reflection is required whenever an interface of sufficient velocity difference is encountered by the ray, or whenever the critical angle is exceeded. At such a boundary the reflected angle θ_R is

$$\theta_R = \pi - \theta_i + 2S_i \pm 2n\pi \tag{30}$$

where n is -1, 0, or 1, so that the condition $0 \leq \theta < 2\pi$ is fulfilled.

Multiple Reflections and Refractions

When the ray is both reflected and refracted at an interface, it splits into two separate rays. To accommodate this fact, and to trace out both ray branches, the information of the angle, position, total travel time and distance to the interface, and approximate amplitudes computed by Fresnel's reflection coefficient at normal incidence must be stored for one ray branch while the information for the other is employed for further ray tracing. This is accomplished by continuing with the reflected ray until it emerges or strikes another interface. In emerging at the surface all the required data, such as distance, total travel time, approximate amplitude, and the depth of penetration of the ray is available. The depth of penetration is found by comparing the depth at all segments of the ray, and retaining the position most distant from the surface. One then returns to the previous interface from which non-critical reflection occurred, and, using the information listed in the last paragraph which has been retained, continues with the refracted ray. As an ensemble of information can be retained from pre-

vious reflections, one is able to accommodate as many multiple reflections as desired.

The multiple reflections may be accommodated in the following manner. Consider arrays of any arbitrary number of elements for each parameter needed to construct a ray, say

$$H(i),V(i),\theta(i),TT(i),TL(i), \text{ and } A(i), \tag{31}$$

where H is horizontal position, V is vertical position, θ is angle of the first refracted segment, TT is travel time to the interface under consideration, TL is total path length to the interface, and A is amplitude; the index i in each value in (31) represents the reflection number. One starts at the inception of a ray with $i=1$. At the first non-critical reflection, the refracted values are stored under the index $i=1$, and the reflected under the index $i=2$; similarly, if another interface with non-critical reflection is encountered, the refracted ray is given the index $i=2$, while the ref. ay is given the index $i=3$, and so on. For each non-critical reflected ray the index is incremented by 1, while the corresponding refracted ray is referred to by the previous index. It is seen that any arbitrary number of reflected rays can be accommodated and traced by incrementing indices in this manner. After any given reflected ray has reached its destination the index i is reduced by 1, and the previously refracted ray is then continued by using the initial conditions of the parameters listed in (31) with the next lower index number. It can be seen that all refracted rays will be traced out until the index reverts to $i=1$, which is the index number of the incident ray. Also, because previously refracted rays may subsequently

encounter a new set of multiple reflections, all multiple reflections of the originally reflected or refracted rays will be accommodated.

Interface Locations

Because interface locations are so critical in affecting the location of the emergence of a ray, a means was developed of precisely locating the interface and extending or retracting the ray to terminate precisely on the interface. This is accomplished in the following manner. The location of a point is found in the square defined by four matrix elements and located on the interface. The horizontal and vertical distance from the ray segment head end to this point is determined. Knowing the slope of the interface, the angle of the ray, and the horizontal and vertical distances to the defined point, a series of computations in which the law of sines is invoked is then made. Many different angular conditions are involved as a result of combinations of ray angle, slope, and positive or negative horizontal distances. For details, one can consult the subroutine GINMAD in the Appendix, where the computations are given in Fortran.

Computer Programs

Three main programs have been developed. The first is for two-dimensional flat earth, the second for two-dimensional spherical earth, and the third for three dimensions. All have been written in the most general computer language, Fortran. The version is Fortran IV-G, each program of which has been compiled and run on the IBM 360/67 computer

of The University of Michigan. The programs include plotting instructions which have been used for the illustrations.

Subroutines have been developed to precisely reflect at an interface, to extend the ray precisely to the surface, to compute Snell's law for any incident angle across an interface of any slope, to increment initial horizontal and vertical positions and angles, and to compute reference travel times for each ray making up an onsetting plane wave.

The indexing for the spherically symmetric case has been made compatible with that for velocity distributions with a rectangular reference system, in such a manner that the data for horizontal layering structures can be read in by a Fortran Namelist. In this way anomalous regions which are very heterogeneous can be included.

The programs have been made general, so that a different number of samples and size of sampling interval, number of multiple reflections, incremented or changed output, number of initializations, etc., can be entered without recompiling the programs. Thirty to forty values are read from Fortran Namelists; the values can be changed individually.

Each ray increment involves calling trigonometric functions approximately 7 to 25 times (usually about 15 times). For an indication of the time required, see Figure 7. The computation for Figure 7, including loading, 20 travel-time printouts, and plotting instructions, required 10 seconds of IBM 360/67 computer time. It is felt that this is reasonable and economical time for such ray tracing.

The three main programs and their subroutines are listed in the Appendix.

Results

Accuracy

The method of ray simulation was developed primarily with synthetic data. First very simple data were used for testing and developmental purposes. As the work progressed, it was felt that a test was necessary to determine its accuracy.

For this reason the method for tracing rays in spherically symmetric earth was attempted. For at least 60 years the earth has been investigated as a spherically symmetric structure in which the velocity gradient everywhere points directly at the center of the earth. Much of seismology has been concerned with this model. Because of the continued investigation of this model over most of the world for such a long period of time, many travel times from individual disturbances at many distances have been recorded.

From these travel times a velocity-depth distribution of the earth has been inferred. This distribution is one of the fundamental problems of seismology, and most seismologists have been concerned with it.

Recently Herrin, et al. (1968) published the 1968 Seismological Tables for P Phases in a special issue of the Bulletin of the Seismological Society of America (BSSA). These tables represented the latest refinements of the contributions of the science of seismology. In this special issue a velocity vs. depth distribution of the mantle was given. As both the velocity distribution and travel times were given in this special issue of BSSA, a model was available to test this method of ray tracing.

As one might expect, the ray tracing method was initially shown to be inaccurate in the first tests. Two improvements were made in the technique as it then existed. The first was to take the incident velocity at the midponnt of the incident ray segment and the emergent velocity at the midpoint of the emergent ray segment. The second change was to repeat the computation of Snell's law, as explained in the two-dimensional modelling section, until the proper emerging velocity was found. It was found with a testing program that the proper value of emerging velocity was approached in a damped, oscillating fashion. It was then a simple expedient to compare successive values until the difference between them was less than an arbitrary value.

This latter change, for which the need was not formerly apprehended, was the key to achieving as much accuracy as desired, without reducing the length of the ray segments to require an uneconomically large amount of computation. As Snell's law is the basis of both this method and that used for the 1968 P-Phase Tables (Engdahl, et al. 1968) the two methods should result in identical relationships between velocity distribution and travel times. Differences should be due only to the size of the sampling intervals, length of the incremented segments, and method of interpolation.

The equivalence between the two methods has been found. As the sampling interval and segment length were reduced, the travel times shown in the 1968 P-Phase Tables have been approached more and more closely when using the same velocity distribution.

The accuracy is illustrated in Figure 2, where the velocity distribution used for the 1968 P-Phase Tables was approximated with samples taken at 15 km intervals with 9 km segments. Figure 2 is a reproduction of a computer output using this ray tracing method and also shows corresponding distances and travel times from the 1968 P-Phase Tables. The average difference between the two methods in travel times for the five P-rays shown in Figure 1 is .252 seconds, or an average difference in travel times between methods of 1/100 of 1%. As the standard deviation of the P-Phase Tables is 1 second, .252 seconds is considered satisfactory. Every indication is that the travel time vs. velocity distribution values could be made closer by further reduction of the sampling interval and segment length.

Plane Waves

Figures 3 and 4 were drawn to investigate the simulation of a plane wave. The velocity structure is based upon a hypothetical velocity distribution within a cross section from the Pacific Ocean across the Coast Ranges, Central Valley of California, the Sierras, and part of Nevada. Figure 3 represents a PcP wave from $\Delta = 30°$ and Figure 4 a P wave from the same distance. The simple subroutine Planit was used as a reference for travel time across the wavefront. These figures were obtained in May 1969 as an aid in Charles G. Bufe's investigation of PcP waves (1969). Bufe was able to estimate PcP-P travel-time differences, and anticipate arrival anomalies.

Spherical Earth

Figures 5 through 8 illustrate two-dimensional seismic ray tracings of an entire cross-section through the earth. In Figure 5 only a single reflection was allowed, enabling one to obtain PcP, PKKP, PKIKKIKP, and P rays. Had no reflection been allowed below the critical angle of incidence, only the PKP, PKIKP, and P would have been drawn in Figure 5. With four reflections allowed one can generate PKP type waves up to PKKKKP, as shown in Figure 6.

In Figure 7 a source at 700 km depth was simulated. Rays were traced from this source at 18° intervals, and each ray traced independently of the others. Note that the second K leg of the initial 18° ray is overtraced by the second K leg of its symmetric counterpart the 360° - 18°, or 342°, ray. This overtrace indicates high positional accuracy, in that Δ and travel times differ by less than .01% between symmetric rays.

Three Dimensional

Illustrations of computer plots in three dimensions are shown in Figures 8 through 11. To clearly show the paths of the rays in three dimensions a computer program was devised to show a perspective and three orthogonal projections. Figures 8 through 11 show a single-point perspective in the upper left portion of the figures, and three orthogonal projections along the principle planes in the remaining portions of the figures. Consider that the (x,y) plane represents the surface of the earth, and the positive z-axis corresponds to depth in the earth.

In Figure 8 and Figure 9 a velocity distribution increasing with depth and having zero slope everywhere with respect to the surface is shown. In Figure 8 cones of rays from a point source at the surface are shown; rays of a sufficiently large initial angle undergo critical reflection and return to the surface (x,y plane). In Figure 9 a line source at depth is shown. Points are chosen along the line, and fans of rays are propagated toward the surface. It should be pointed out here that any type of initializing rays may be chosen. For example, in Figure 9 the points of origin along the line could have been made much closer together, and the angular extent of the fan decreased, with any choice of angles between individual rays chosen. As described in the section on the computer programs, the values for the increments of these points and angles can be entered in the Namelist.

Figures 10 and 11 illustrate an oblique gradient with respect to the z-axis. The mathematical distribution of the velocities in Figure 10 is

$$V = 6.0 + X + 2Y + 3Z \qquad (32)$$

These illustrative figures make use of velocity values which exceed normal earth velocities. These velocities were used for the purposes of demonstrating the capability of tracing such a distribution. In Figure 11 a sampled layer extending from a level corresponding to 40 km depth to 190 km depth is placed within the field described for Figure 9. The gradient is parallel to the z-axis within this slab, and from 140 km to 190 km depth the velocities are constant. As shown in Figure 10, one can define regions in three-dimensional space which can be either analytical mathematical functions or sampled data. The number, sizes, or shapes of the regions are not restricted.

Potential Application to the Seafloor Spreading Problem

In the theory of seafloor spreading, the earthquake zones are associated with upward welling zones, such as the Mid-Atlantic Ridge, where surface material is being produced, and the boundary between large lithospheric plates as in the Circum-Pacific zone.

The problem of these boundaries, one of which is impinging on the other, is treated by Isacks, Oliver, and Sykes (1968). They postulate that one plate underthrusts the other, particularly in island arc regions. At this boundary the lithosphere (the upper 100 km of the earth's surface) of the oceanic plate is bent downward into the underlying asthenosphere of the continental plate. The precise geometry in which the two plates join is not known, and differs between regions. Isacks, Oliver and Sykes (1968) postulated several configurations as a function of local conditions and rate of underthrusting. Mitronovas, Isacks, and Seeber (1969) have investigated this problem in the Tonga Islands arc. Murdock (1969) has discussed velocity distribution under the Aleutian Region. Carder, et al. (1967) have documented travel-time anomalies from the LONGSHOT explosion.

Figures 12 through 15 were drawn with an arbitrarily-chosen geometry based on the models postulated. In the geometry chosen the 6.75 km/sec layer is taken as the topmost layer which bends downward and the surficial 6.00 km/sec layer is unaffected by bending. These figures show different earthquake locations and the directions of seismic waves propagating from them. Two of the earthquakes are directly above and two directly beneath the upper boundary of the underthrusting lithosphere. The two earthquakes

above the boundary are located in regions where volcanic activity and minor seismic activity is indicated (Mitronovas, Isacks, and Seeber, 1969). The two earthquakes beneath the boundary are located where most seismic stress and the highest probability of earthquakes are thought to occur. The rays for these plots were generated at 4° radial increments from a point source.

Although the frequency of earthquakes with sources above is less than the frequency of those within the underthrusting lithosphere, both source regions have been shown for contrast. This contrast arises from trapped waves within the low velocity layer when the source is within the layer. Rays from within the layer critically reflect at the interface at angles greater than 55° from the normal, to the interface. Rays from sources outside the low velocity layer can be critically reflected only when the inclination of the interface through which the rays have entered is oriented differently from the interface through which they would have otherwise emerged. Parallel interfaces bounding a low velocity layer cannot trap rays which enter from outside the layer. In the model used for Figures 12 through 15 critical reflection from externally generated rays can only occur where the interface of the low-velocity layer is curved, as shown by the one critically reflected ray in Figure 12.

The effects of the low-velocity layer are clearly seen in Figures 12 through 15. Shadow zones are found at the surface when the source is above the layer. These occur because the rays penetrating the layer are bent toward the normal of the interface. These rays are displaced from the paths they wouldhave taken in the absence of the low-velocity layer,

and they emerge at a greater distance from the source than would otherwise be the case. From sources within the low-velocity zone, the influence of critical reflections is shown in Figures 14 and 15. Three separate groups of rays can be seen propagating up the low-velocity layer: those initially reflecting from the upper interface, those which do not touch either the upper or lower interface, and those initially reflecting from the lower interface.

Travel times from each of the four sources are shown in Figures 16 through 19. In Figures 16 and 17, the focal depths are 50 km and 100 km respectively, with sources above the layer. The shadow zones caused by the ray displacements are evident. Travel-time anomalies above 2 seconds occur near the limit of the shadow zones nearest the source.

Travel times for sources within the downward bending layer are shown in Figures 18 and 19, where the focal depths are 60 and 110 km respectively. Critical reflections occur with a large angular extent from the source. The three separate arrival sequences are produced by the three groups of rays described above. The three separate legs of the travel-time curves occur at distances approximately the same as those where the shadow zones are produced for the sources above the layer as shown in Figures 17 and 18.

These results indicate that large effects upon travel times are found within 50 km of the point where the underthrusting lithosphere starts to bend downward from the unperturbed oceanic plate. A pertinent investigation of seismic propagation in the Tonga Islands arc (Mictonovas, et al. 1969) was conducted with five seismometer locations

spaced at distances of 125 km to 200 km from this point toward the underthrusting lithosphere. The nearest seismometer location toward the oceanic plate was approximately 220 km distant on Raratonga in the Cook Islands. Unfortunately, the location of the seismometers were dictated by the positions of the available islands, and were not within the region in which ray tracing shows shadow zones and multiple arrivals.

To investigate the travel times beyond the region shown in Figures 12 through 15, the spherical earth program was attached as a subroutine to the flat earth program. Many rays exceeded the boundaries shown in Figures 12 through 15 without reaching the surface. These rays were used as sources for the spherical earth program. This was accomplished by saving the location, angle, travel time, and distance travelled as an input to the spherical earth subroutine. Travel times out to approximately 60° were found. These closely paralleled the 1968 P-Phase Tables for corresponding depths of focus. The anomalies found were a maximum of .9 seconds, and were at the maximum in the region near 25°. No conclusions were drawn from these results.

Figures 12 through 19 illustrate the diagnostic capability of the method presented in this study. The travel times are particularly revealing at distances within 50 km of the point at which the lithosphere starts to bend downward. They depend upon the location of the source, vary with direction, and produce distinctive features in the ray diagrams and travel-time curves. Given a set of seismograms at suitable distances from the earthquake in a continental plate boundary area, an investigation of the velocity distribution and source depth can be aided

with this method. A more detailed investigation would also include azimuthal effects, which could be found by projecting the given model on cross-sections at various angles with the cross-section shown.

It is not the purpose of this study to investigate a portion of the earth in detail, but rather to present a method to aid in such investigations. For example the travel times and anomalies found with curved earth out to 60° were not pursued as to their meaning, but rather a method to obtain them using a spherical earth subroutine was demonstrated. Also, the model chosen for the underthrusting lithosphere did not include the 6.00 km/sec layer. Many variations of the model would be made during a detailed investigation--variations such as thicknesses of the layers, effects of depth, and, therefore, heat, on the velocity of the layers, angle of downward bending, extent of the downward thrusting layer, effects upon the overlying layers, departures from horizontal layering, and many other variations. It is apparent that such changes can be accommodated by this ray simulation method. The results of these changes are given both visually and numerically.

Conclusions

This study has resulted in an economical and versatile method to aid in seismic analysis. Geometric ray tracing can be performed with models representing any conceivable velocity distribution within the earth. Refractive as well as reflective computations can be performed, including multiple reflections. Travel times and approximate amplitudes can be determined for these velocity distributions.

One of the primary goals of seismology is to model the velocity distributions within the earth. Hypotheses of velocity distributions are continuously being produced by seismologists. An important test for the validity of these models has been developed.

Computer drawn plots and numerical output of travel times have been used to illustrate the capabilities of this simulation method. The travel times can be made arbitrarily close to those published by the Seismological Society of America. This method of geometrical ray tracing performs as accurately and with more versatility than previously published ray tracing techniques.

It is hoped and expected that this technique will be used by the seismological profession and that it will be extended and improved, as any useful, new contribution should.

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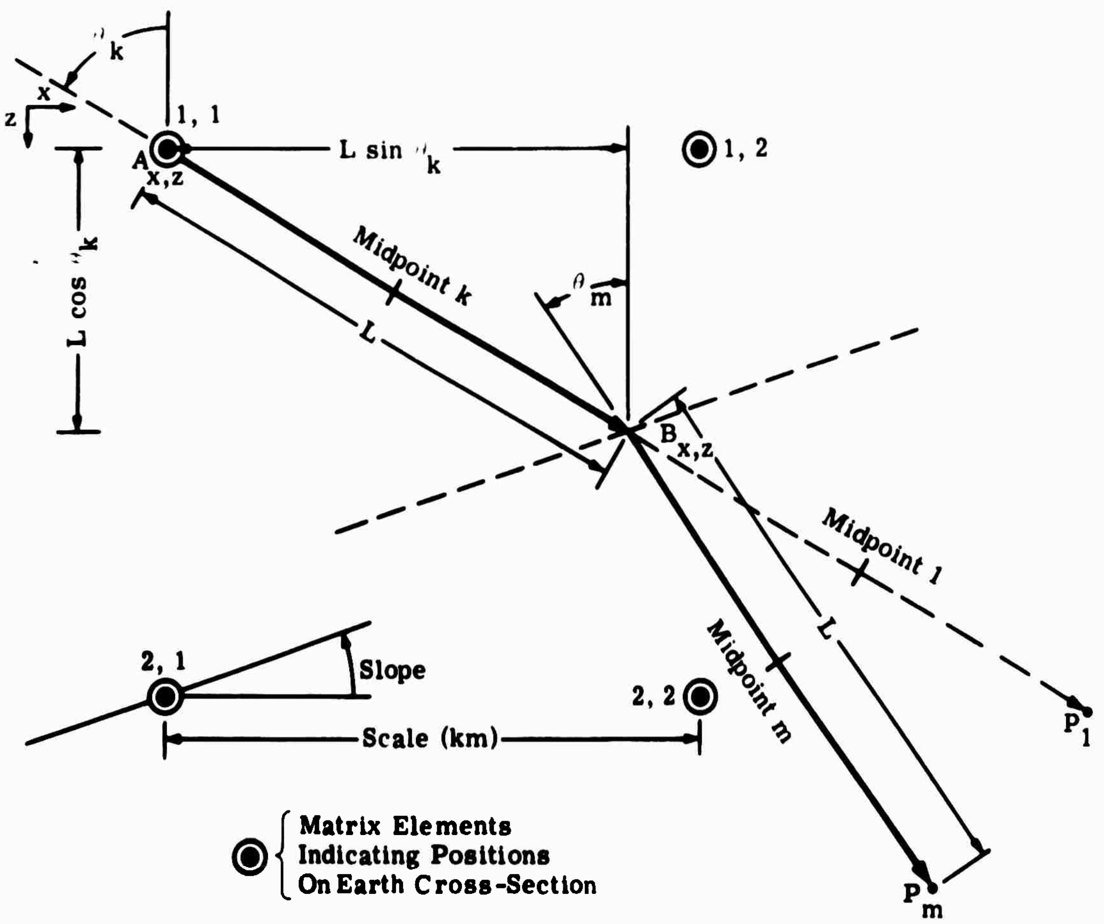

FIGURE 1. REFERENCE FRAME FOR INCREMENTING RAYS. Seismic velocity and slope are associated with each matrix element. Referencing for the slope is shown at element (2, 1). An interface between two velocity layers can be positioned between matrix elements as shown by the dotted line through B. The scale, or sampling interval, is the distance between matrix elements as represented in kilometers. An incident ray segment of length L and initial angle θ_k extends from A to B. Coordinates of B are found by extension from A: $B_x = A_x + L \sin \theta_k$, $B_z = A_z + L \cos \theta_k$. The segment BP_1 is the first "probing" segment to obtain initial values of seismic velocity at midpoint 1. The angle θ_m is computed with Snell's law, using emerging velocity at midpoint 1, incident velocity at midpoint k, and incident angle θ_k. "Probing" segments are recomputed using successive values of θ_m until the absolute value of $(\theta_m - \theta_{m-1})$ is less than a predetermined small value. The velocity and slope associated with the segment BP_m and the last computed value of θ_m are then used as the incident ray parameters, and a subsequent emerging angle computed for the ray segment proceeding from point P_m.

COMPUTER OUTPUT								CORRESPONDING TRAVEL TIMES FROM 1968 P TABLES	
> INITIAL ANGLE, DEPTH, & DISTANCE									
>DELTA	KM	MIN	SEC	TOTSEC	RADIUS	DEPTH	AMPL	MIN	SEC
#> 17.00	0.0	0.0							
> 82.07	9148.83	12.37	21.99	741.99	3970.18	2400.82	0.90	12	21.66
> 20.00	0.0	0.0							
> 71.43	7962.47	11.36	21.62	681.63	4448.43	1922.57	1.04	11	21.49
> 23.00	0.0	0.0							
> 58.41	6510.93	9.94	56.32	596.32	4880.27	1490.73	1.28	9	56.42
> 26.00	0.0	0.0							
> 46.62	5196.48	8.48	29.08	509.08	5243.15	1127.85	1.62	8	29.15
> 29.00	0.0	0.0							
> 33.49	3732.67	6.66	39.86	399.86	5546.36	824.64	2.27	6	40.49

FIGURE 2. COMPUTER OUTPUT USING DIRECT RAY SIMULATING FOR 5 P RAYS. 15 km sampling interval and 9 km incrementing distance per iteration. Travel times from 1968 P-phase tables shown for comparison. Travel times are shown in integer and fraction notation in minutes, seconds corresponding to fraction of the minute, and in total seconds. For example, in the entry for = 82.07°, 21.99 sec = 0.37 min, and travel time = 12 min 21.99 sec.

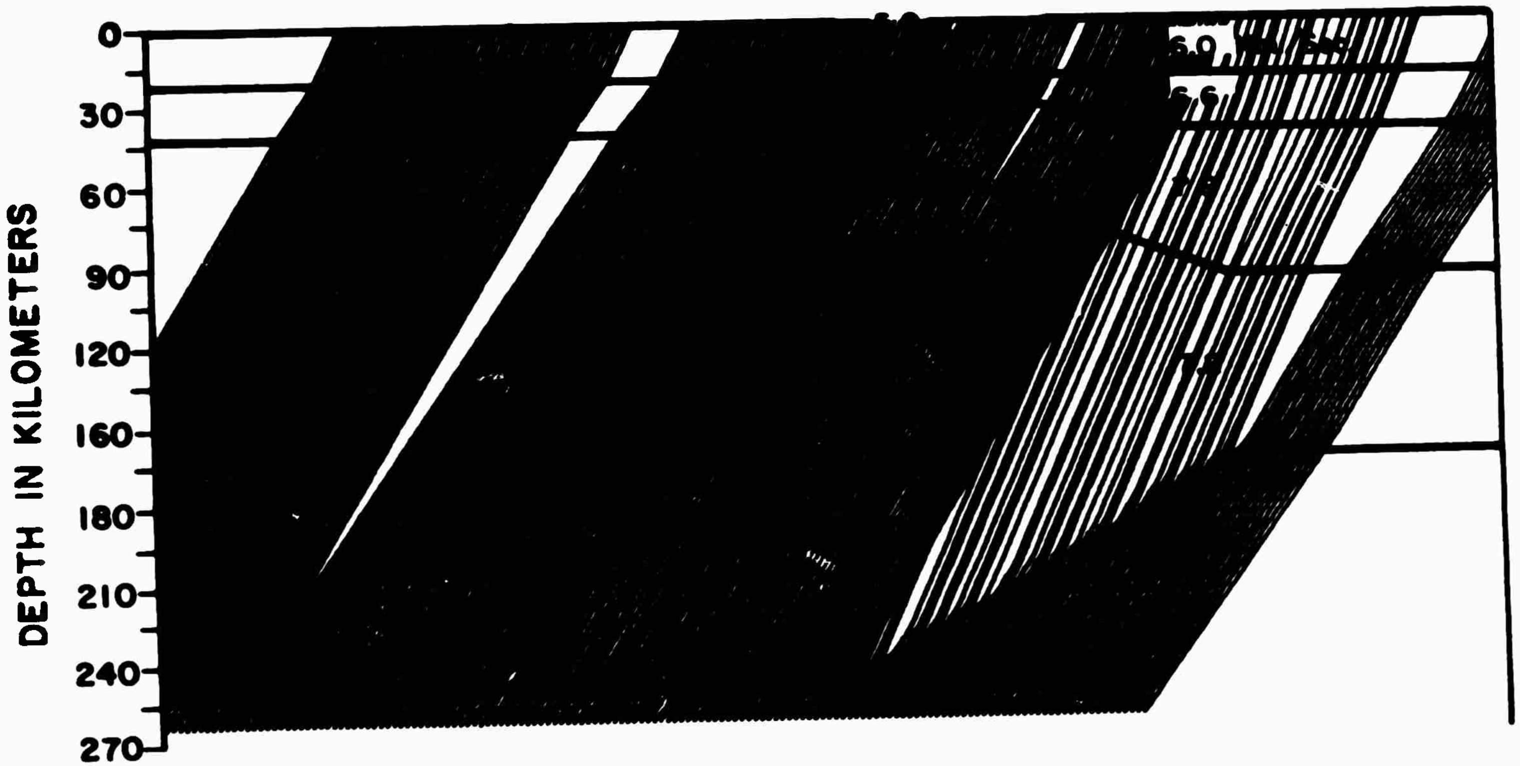

FIGURE 3. PROPAGATION OF P THROUGH HYPOTHETICAL CRUST AND UPPER MANTLE STRUCTURE. Cross-section from Pacific Ocean across central California to Nevada. Plane wave simulation of P-wave from source 30°, or 3,333 km, distance. Emerging rays indicate focussing, shadow zones, and travel time anomalies. From Bufe (1969).

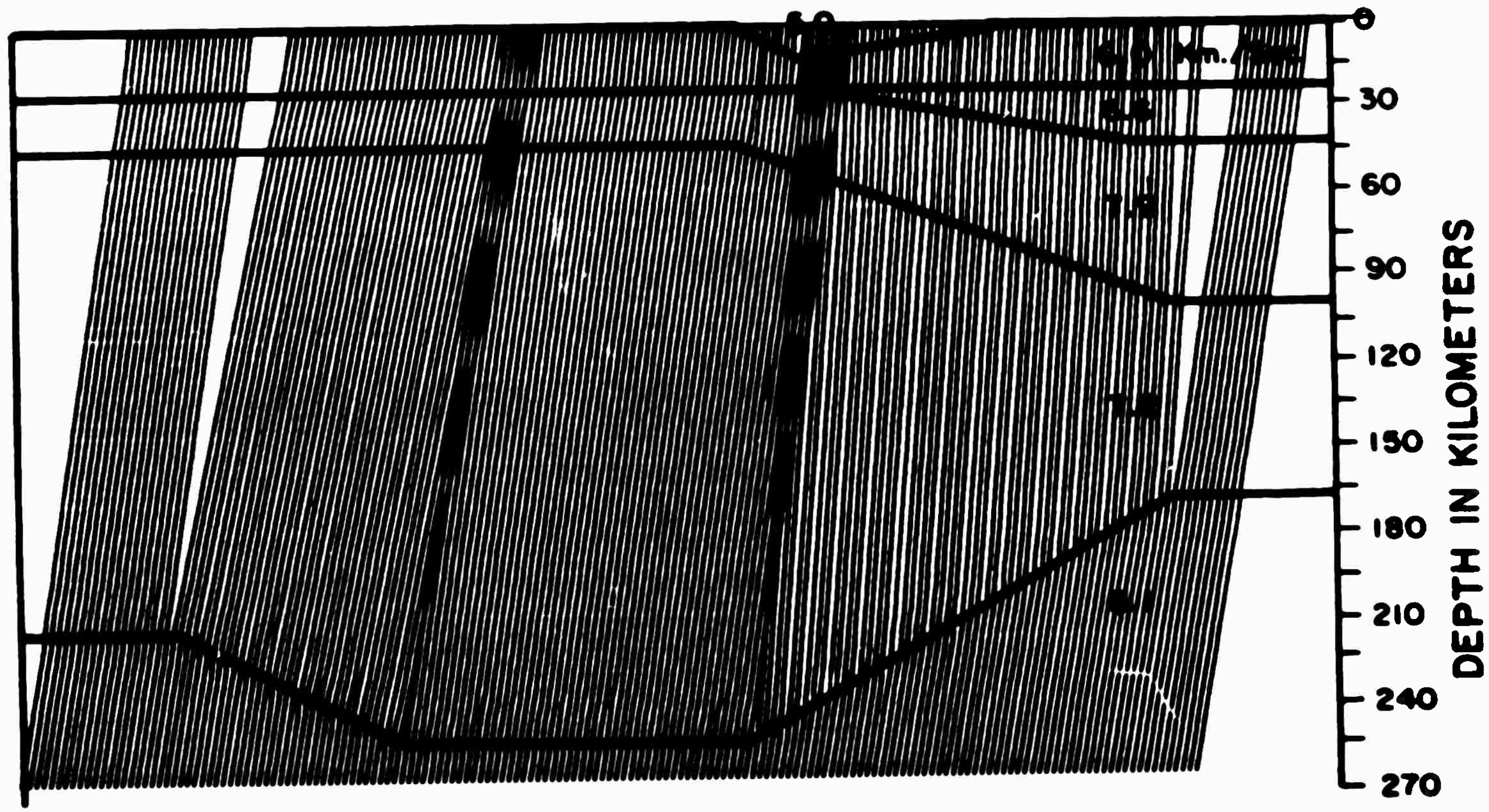

FIGURE 4. PROPAGATION OF PcP THROUGH HYPOTHETICAL CRUST AND UPPER MANTLE STRUCTURE SHOWN IN FIGURE 3. Note significantly different distribution of emerging rays from those in Figure 3. From Bufe (1969).

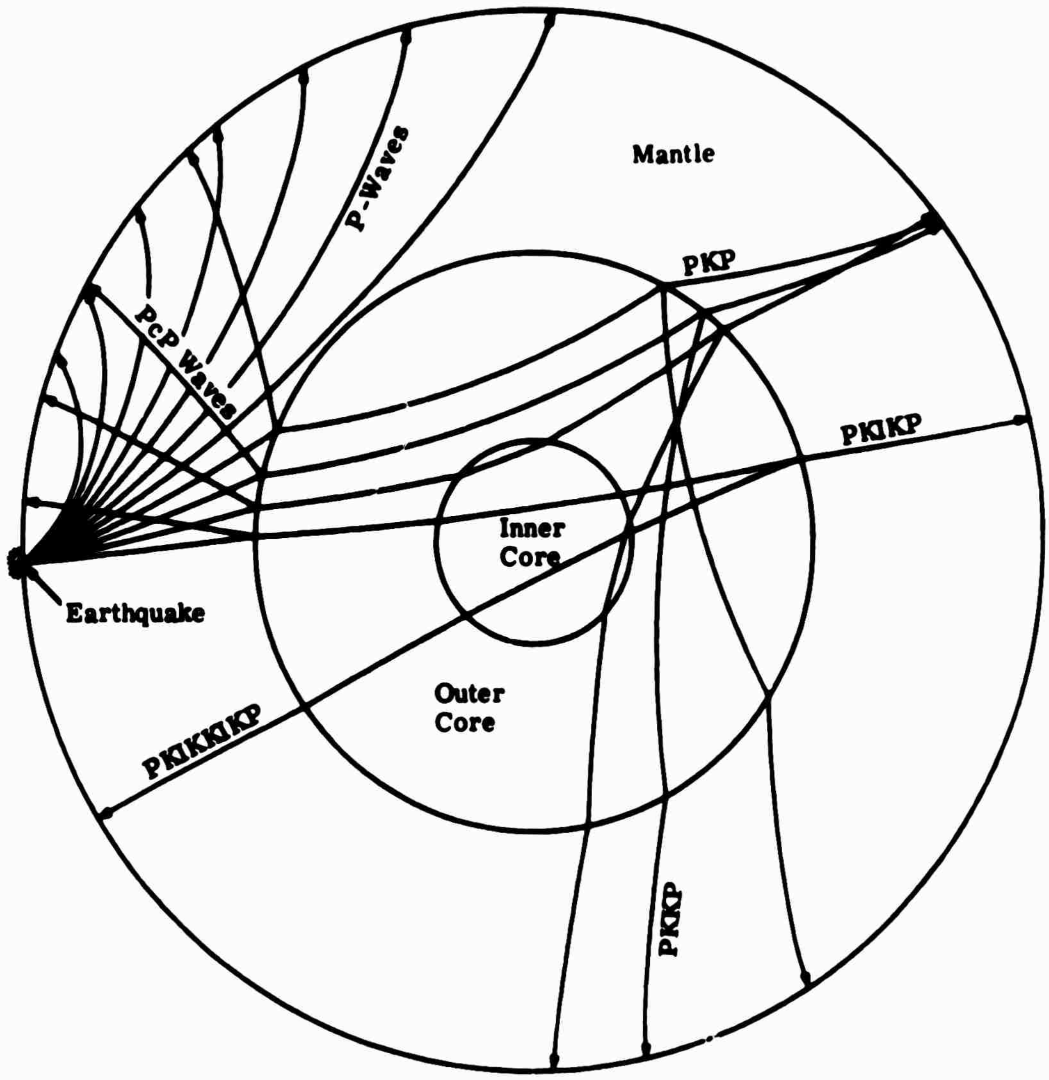

FIGURE 5. PLOTS OF P, PcP, PKP, PKIKP, PKKP, AND PKIKKIKP RAYS GENERATED WITH A SPHERICAL EARTH PROGRAM. Jeffreys-Bullen core and 1968 P-phase mantle velocity distributions. Radial velocity distribution sampled at 70 km intervals. Rays segment lengths 70 km.

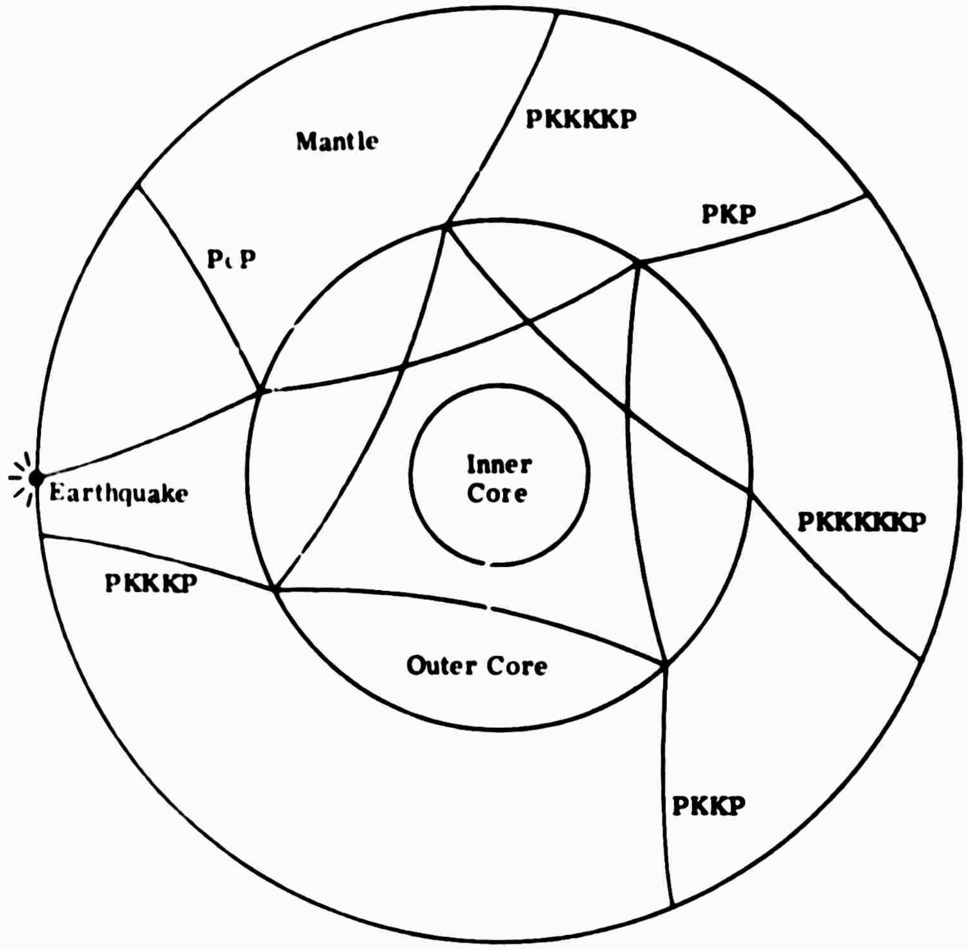

FIGURE 6. MULTIPLE REFLECTIONS. PcP and PKP rays up to PKKKKKP. Four multiple reflections preset into computer program. Any number of multiple reflections can be preset into program.

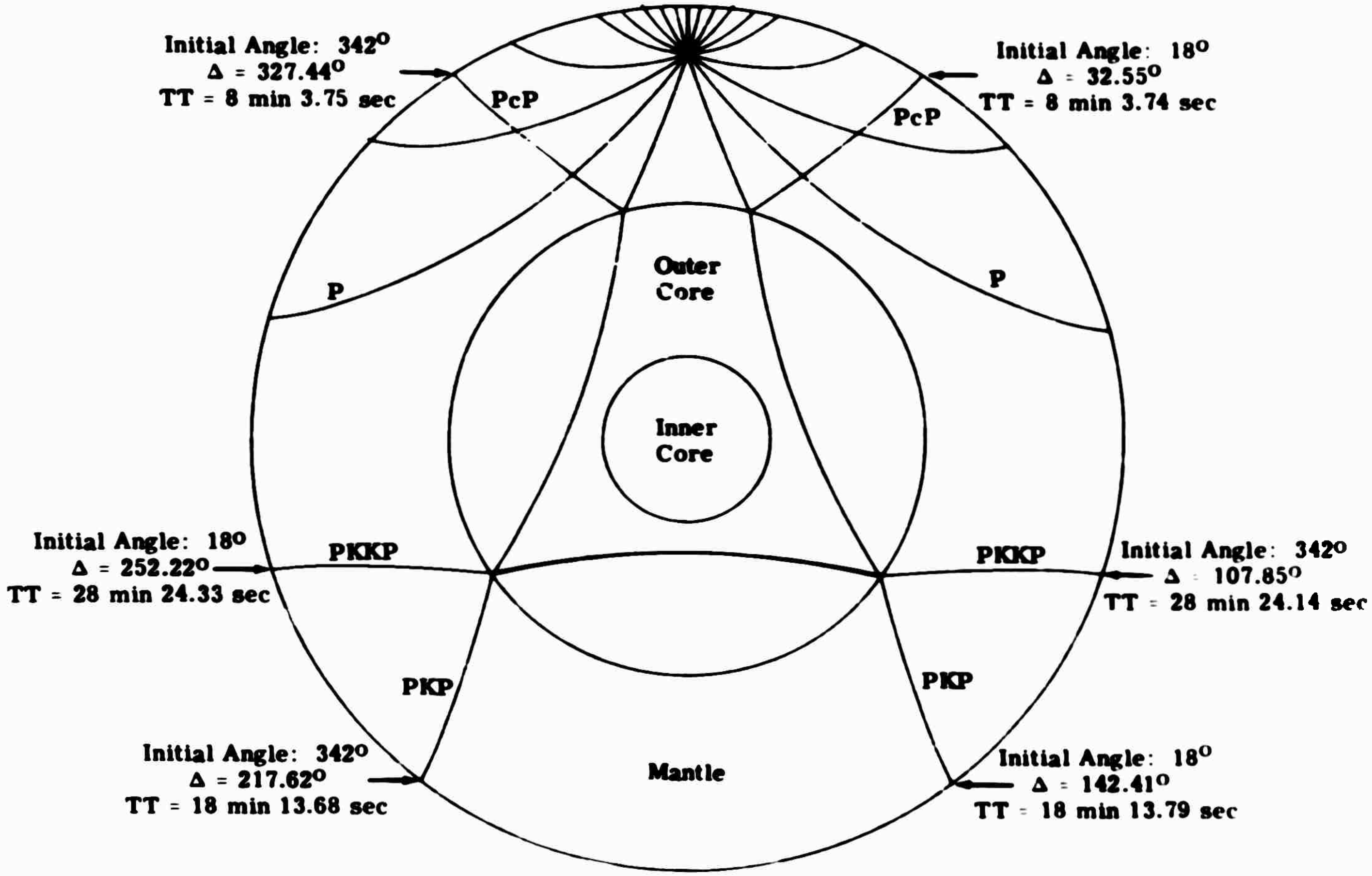

FIGURE 7. UNIFORMITY OF PLOTTING AND TRAVEL TIMES SYMMETRICALLY PLOTTED FROM 700 km DEPTH. Initial angles from source are in increments of 18° from vertical. Computer results for and travel times of emerging rays shown for 18° and 342° rays. Maximum difference in travel time between symmetric rays is 0.11 seconds in PKP rays.

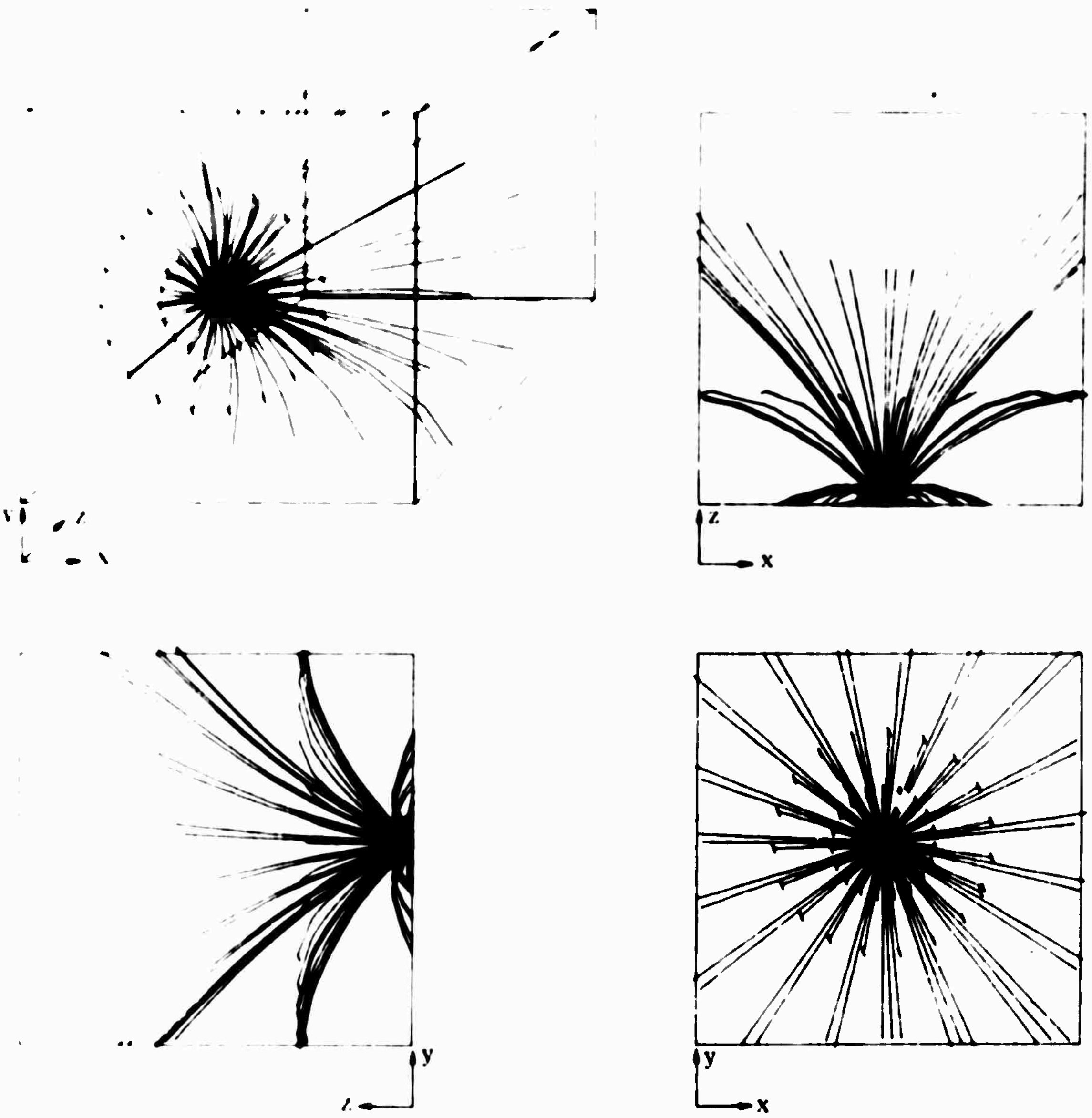

FIGURE 8. THREE DIMENSIONAL RAY TRACING IN A REGION DEFINED BY 30 × 30 × 30 VELOCITY SAMPLES. Constant velocity in planes parallel to the x, y plane, increasing velocity in z-direction for 15 sample points from the x, y plane. Four cones of rays originating from point in the x, y plane. Rays emerging at the x, y plane shown by slash marks with orientation of angle of emergence. Upper left: one-point perspective; remaining views: labelled orthographic projections.

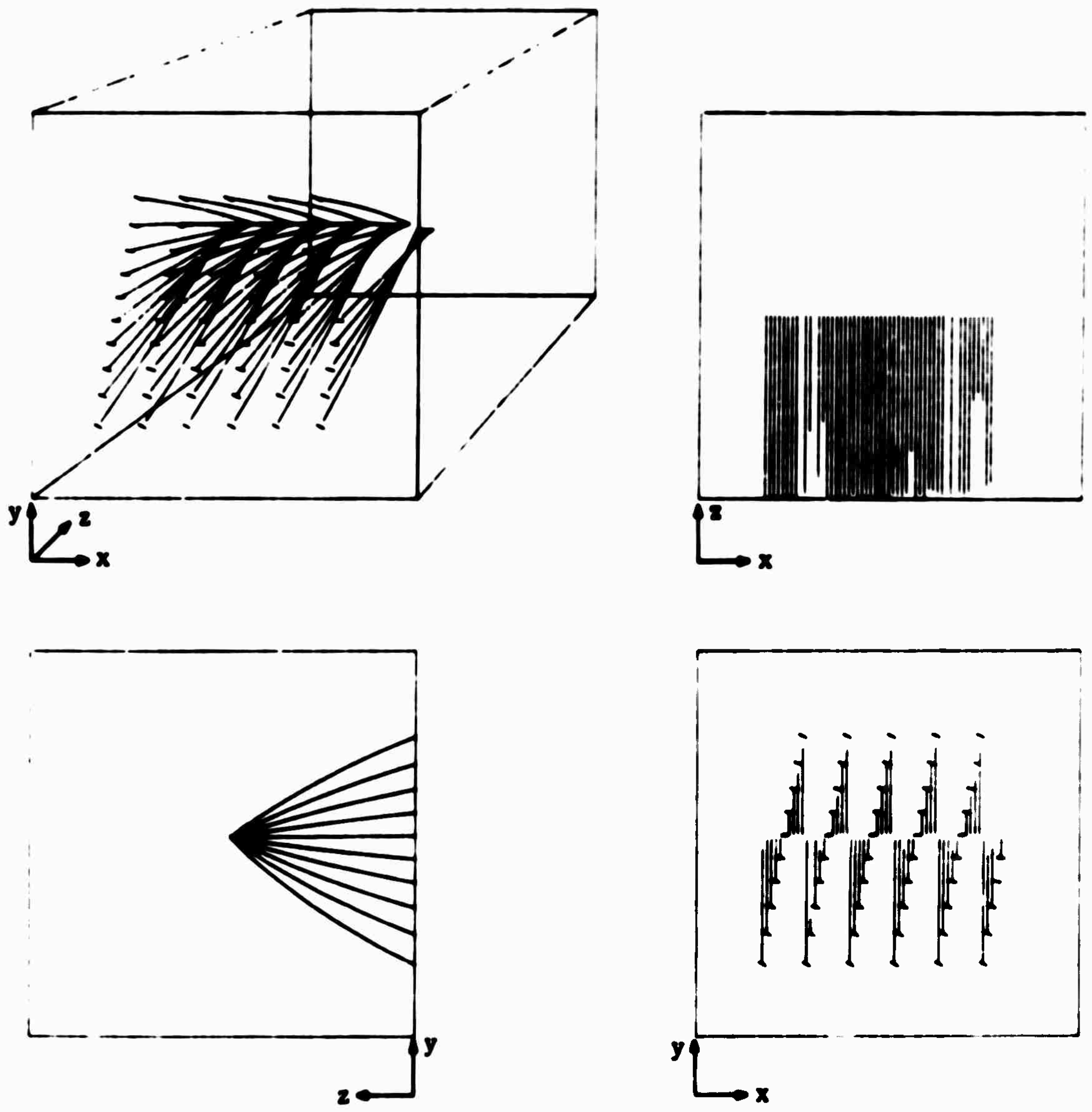

FIGURE 9. LINE SOURCE AT DEPTH FOR THREE-DIMENSIONAL REPRESENTATION AS SHOWN IN FIGURE 8. Plotting defects in upper right (x, z) projection; all lines should extend to x-axis.

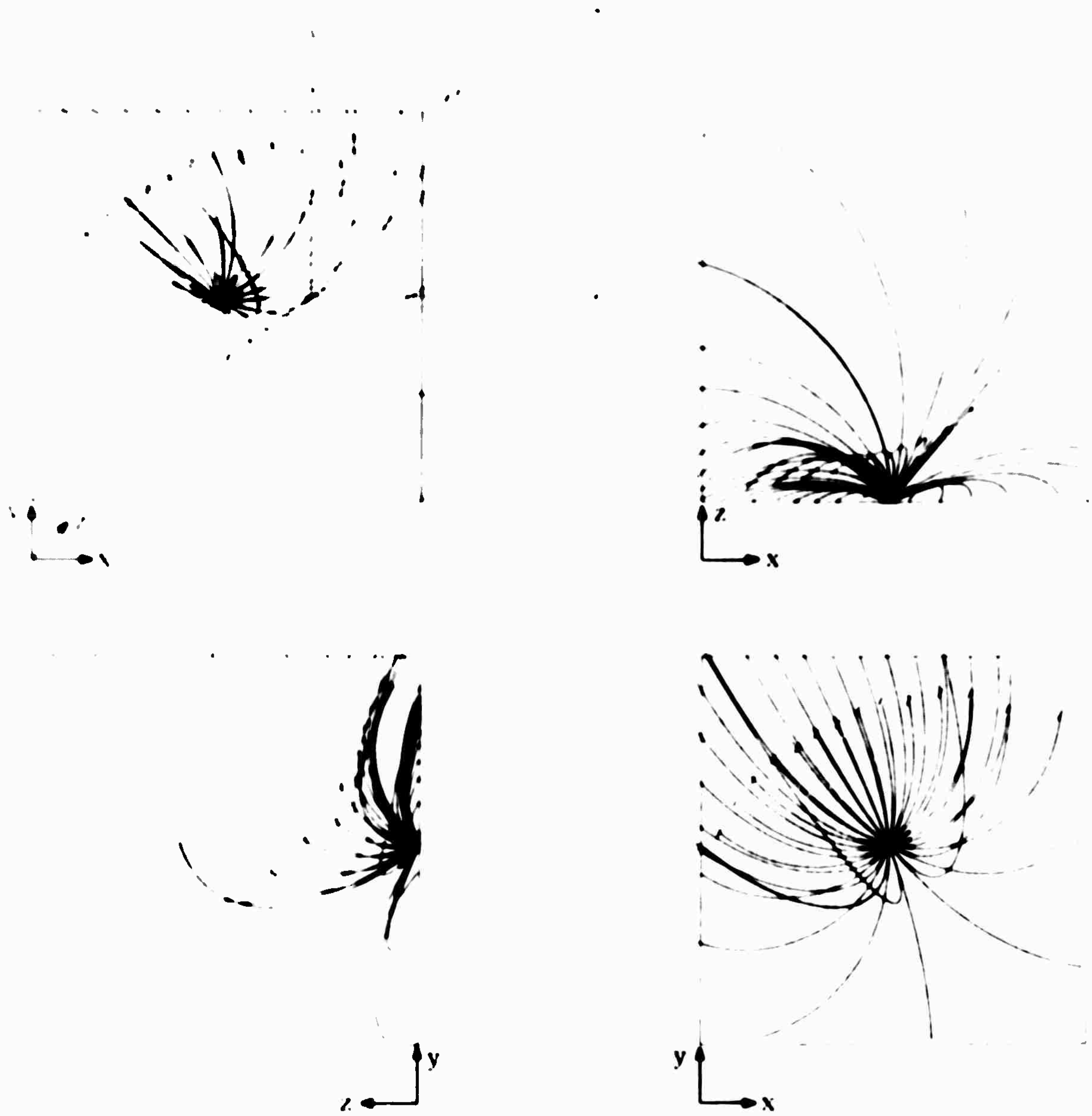

FIGURE 10. THREE-DIMENSIONAL RAY TRACING WITH VELOCITY DISTRIBUTION DEFINED BY CONTINUOUS MATHEMATICAL FUNCTION. Velocity = 4.0 + 0.15x + 0.3y + 0.6z, where volume is defined as in Figure 8.

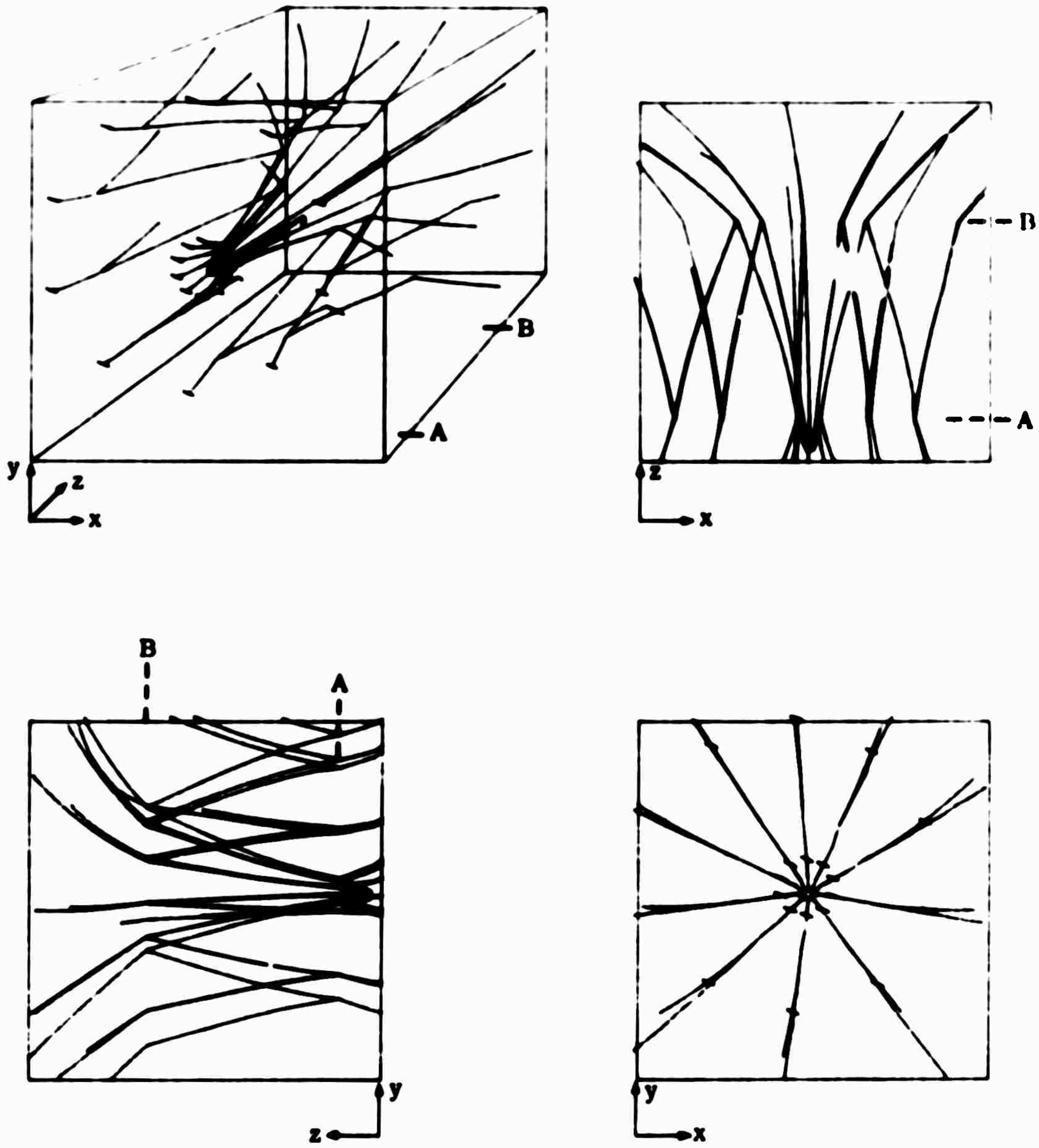

FIGURE 11. THREE-DIMENSIONAL RAY TRACING WITH VELOCITY FUNCTIONS SPECIFIED FOR DIFFERENT REGIONS. Sampled values for intermediate layer between A and B. Velocity = 4.0 + 0.15x + 0.3y + 0.6z for layers above A and below B. Volume is defined as in Figure 8.

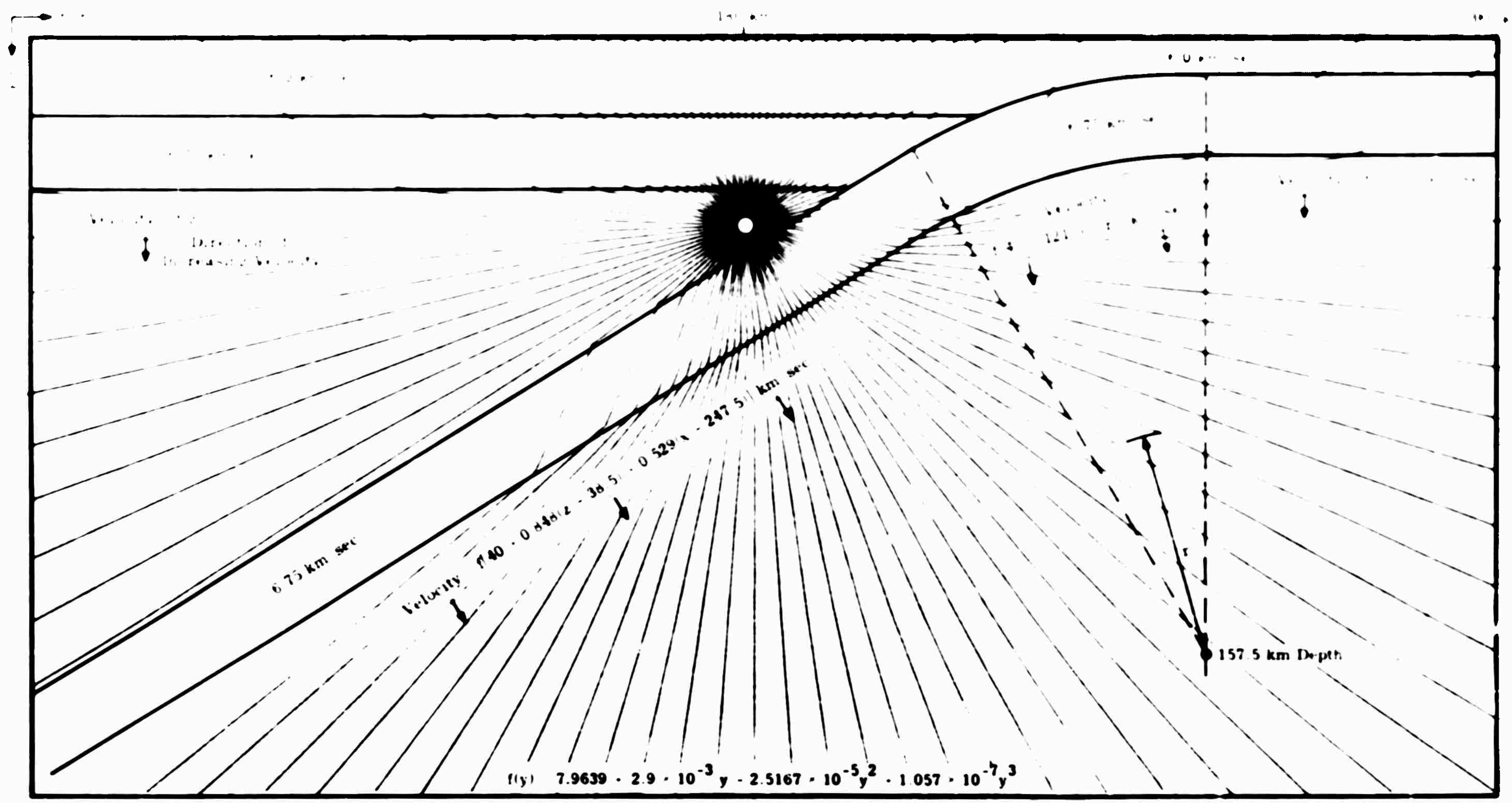

FIGURE 12. RAY PATHS FROM EARTHQUAKE ABOVE UNDERTHRUSTING LITHOSPHERE AT CONTINENTAL PLATE BOUNDARY. Depth of focus: 50 km. Velocity distribution for Figure 12-19 based upon Isacks, Oliver, and Sykes (1968). Travel times for this plot are shown in Figure 16.

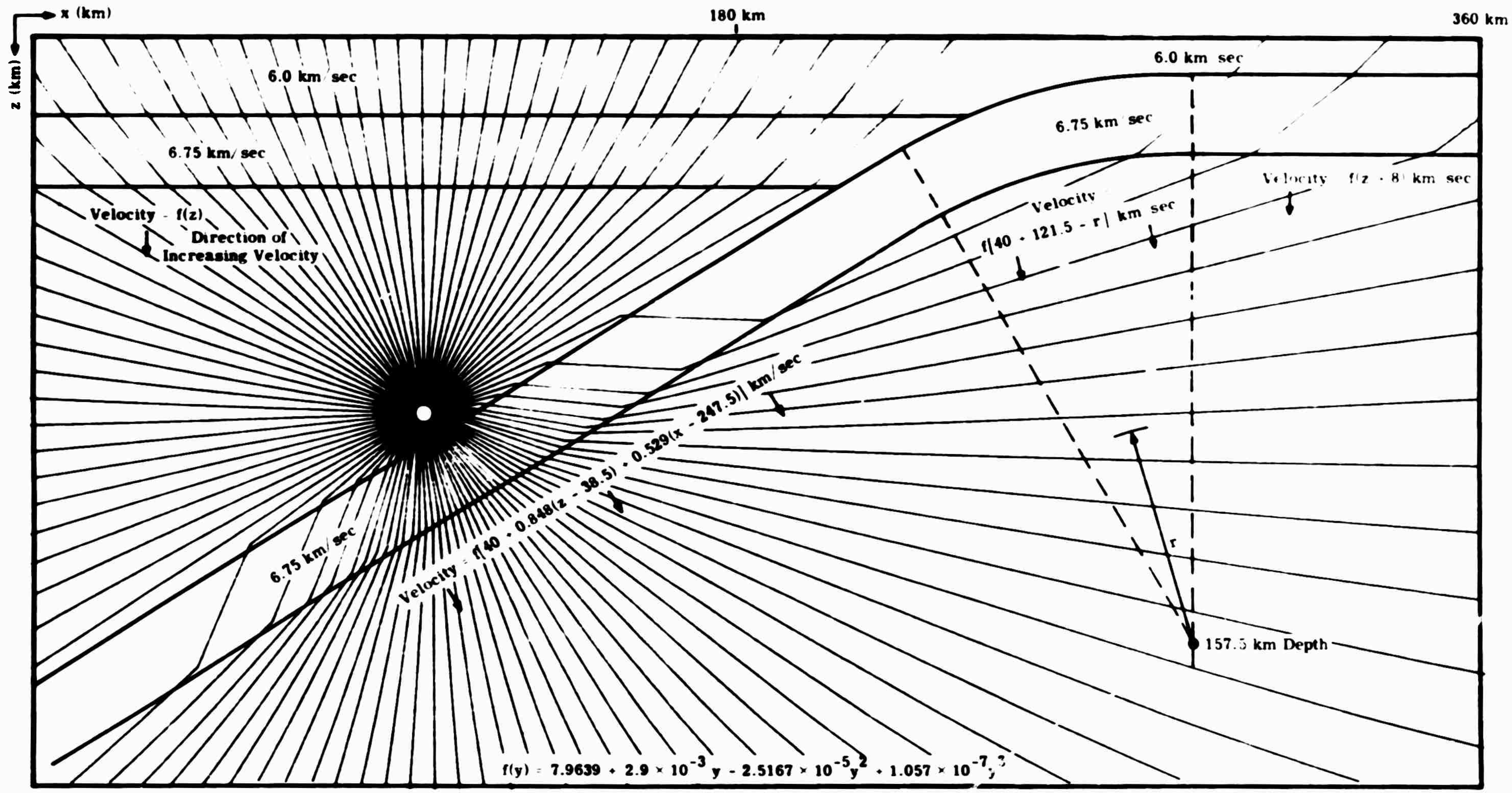

FIGURE 13. RAY PATHS FROM EARTHQUAKE ABOVE UNDERTHRUSTING LITHOSPHERE AT CONTINENTAL PLATE BOUNDARY. Depth of focus: 100 km. Travel times for this plot are shown in Figure 17.

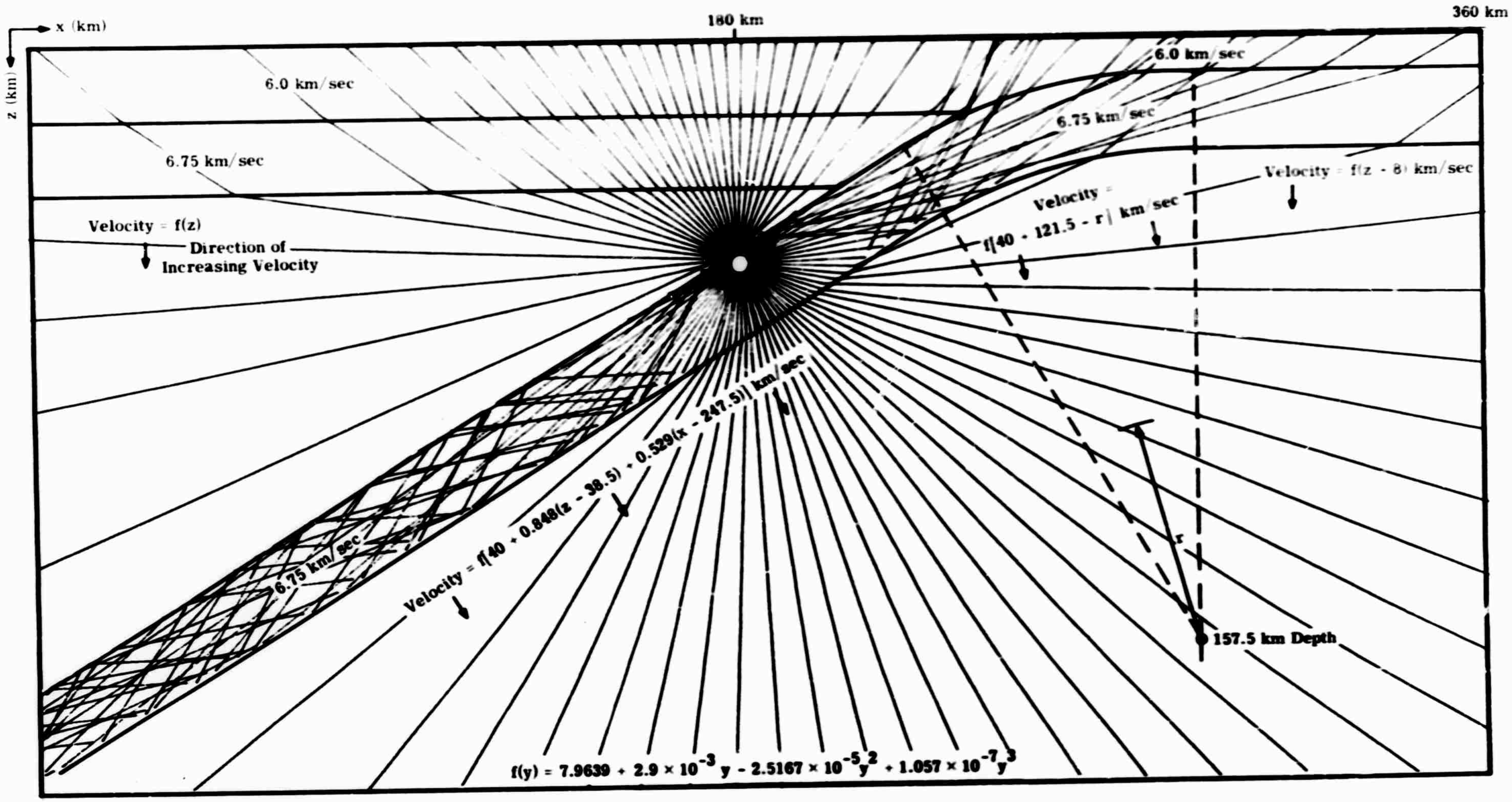

FIGURE 14. RAY PATHS FROM EARTHQUAKE WITHIN UNDERTHRUSTING LITHOSPHERE AT CONTINENTAL PLATE BOUNDARY. Depth of focus: 60 km. Travel times for this plot are shown in Figure 18.

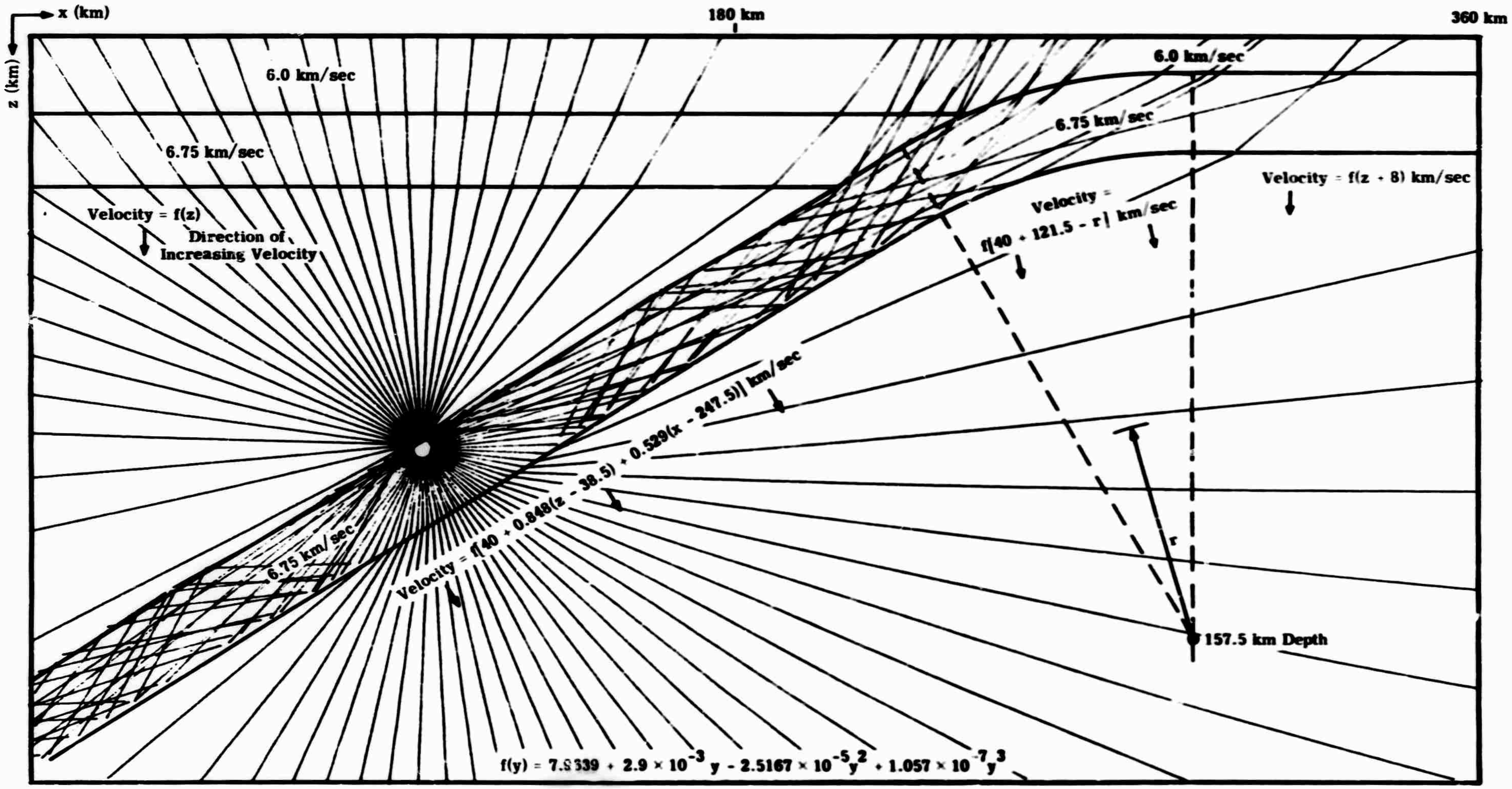

FIGURE 15. RAY PATHS FROM EARTHQUAKE WITHIN UNDERTHRUSTING LITHOSPHERE AT CONTINENTAL PLATE BOUNDARY. Depth of focus: 110 km. Travel times for this plot are shown in Figure 19.

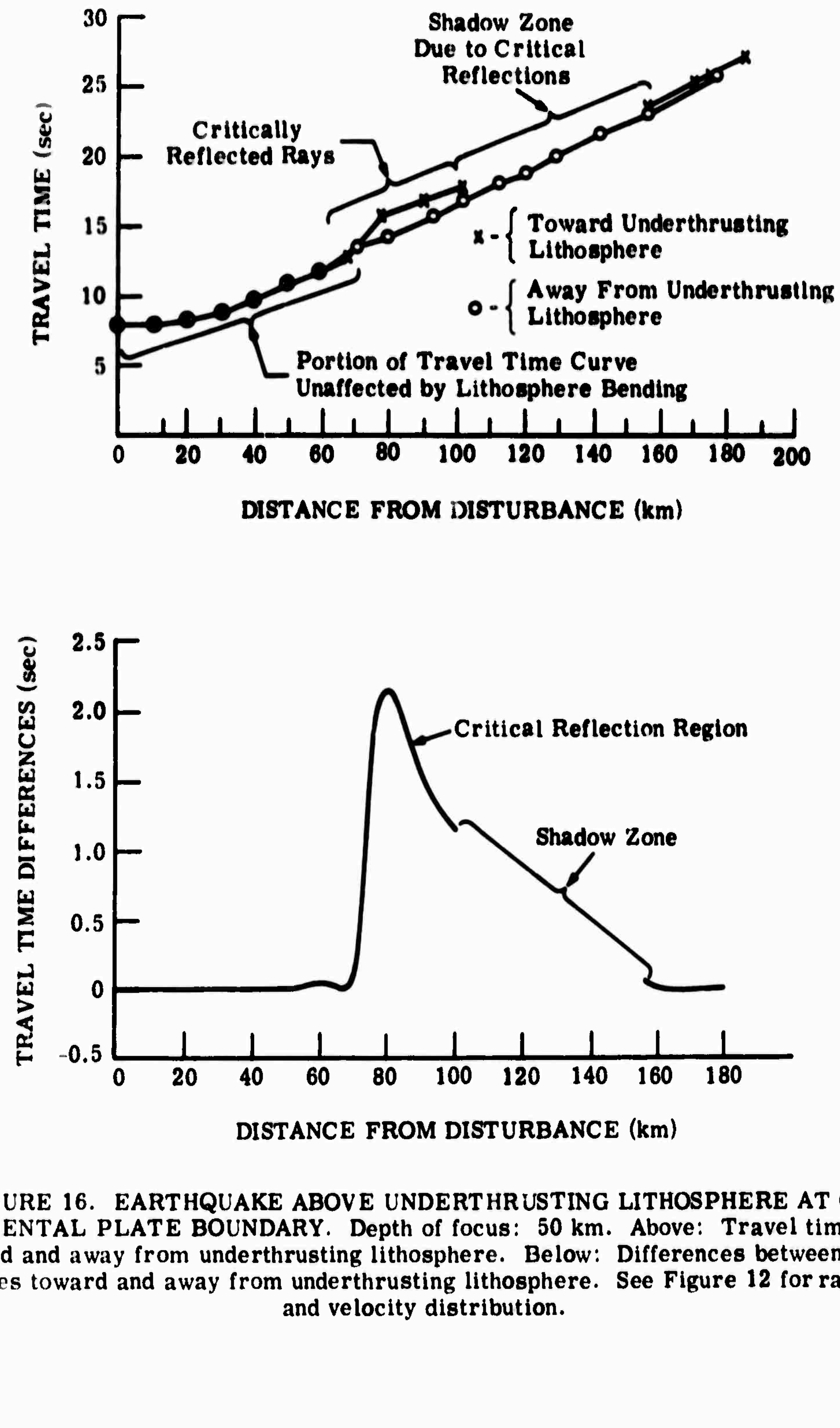

FIGURE 16. EARTHQUAKE ABOVE UNDERTHRUSTING LITHOSPHERE AT CONTINENTAL PLATE BOUNDARY. Depth of focus: 50 km. Above: Travel times toward and away from underthrusting lithosphere. Below: Differences between travel times toward and away from underthrusting lithosphere. See Figure 12 for ray paths and velocity distribution.

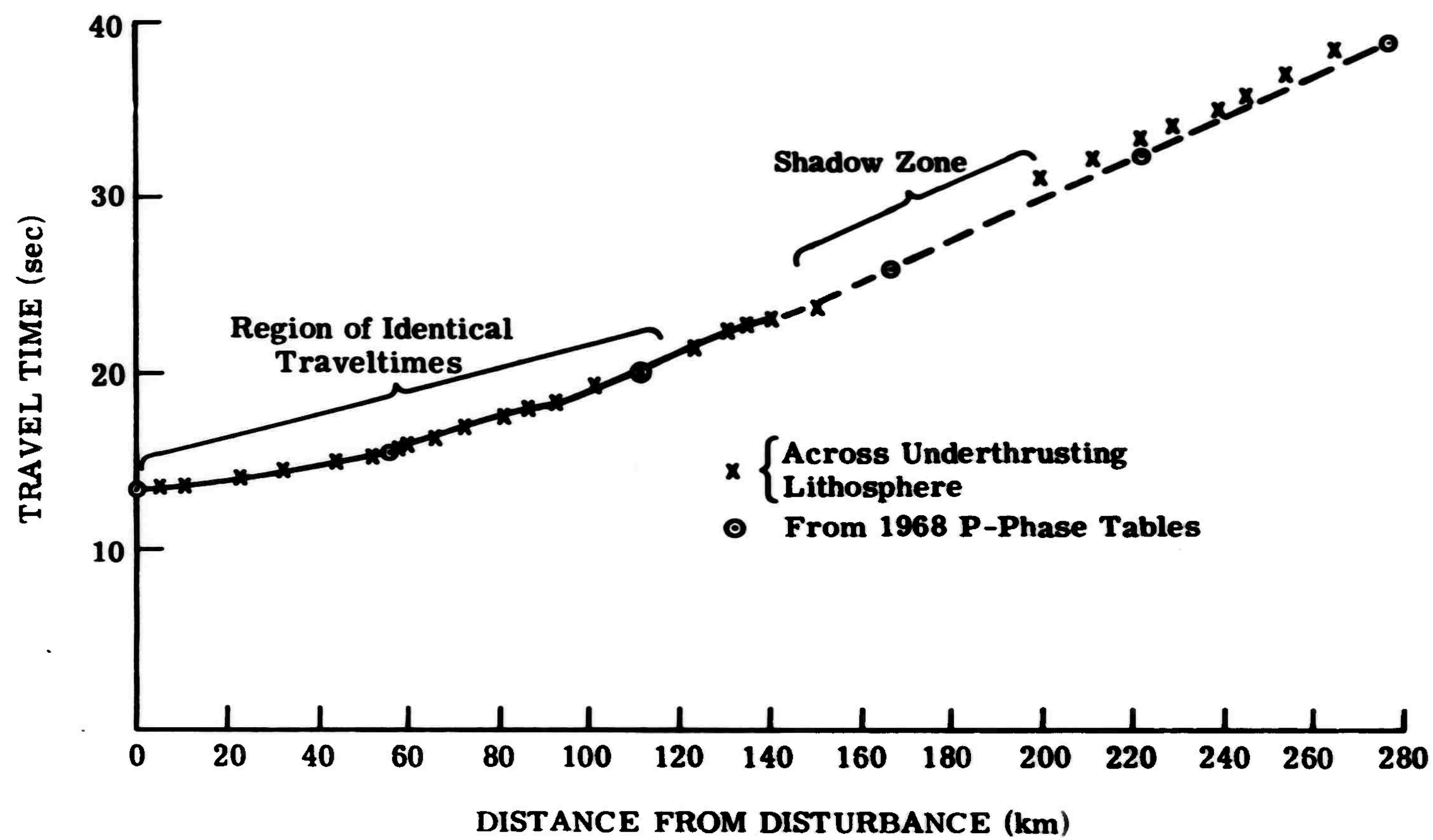

FIGURE 17. EARTHQUAKE ABOVE UNDERTHRUSTING LITHOSPHERE AT CONTINENTAL PLATE BOUNDARY. Depth of focus: 100 km. Travel times from 1968 P-phase tables shown for comparison. See Figure 13 for ray paths and velocity distribution.

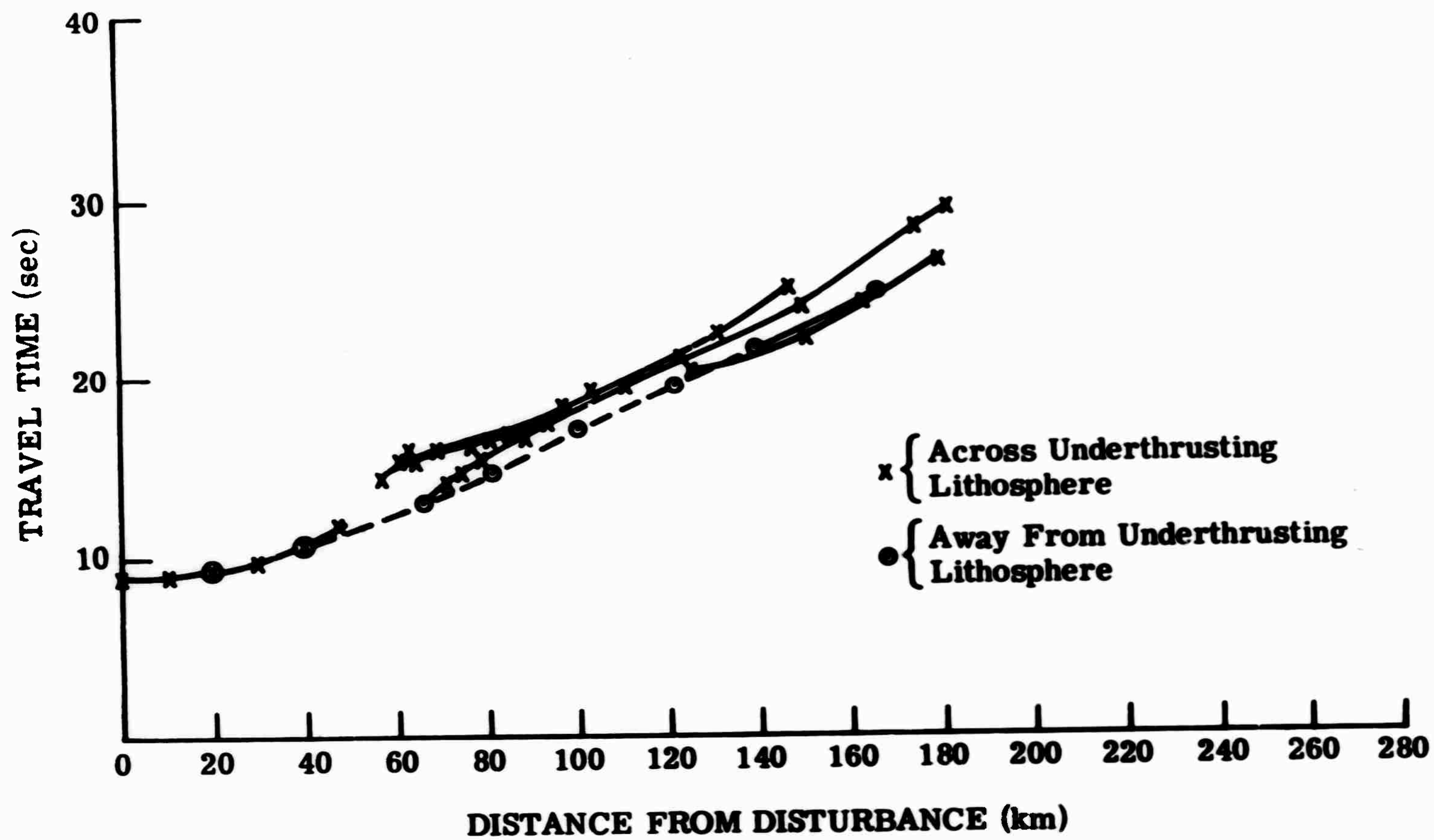

FIGURE 18. EARTHQUAKE WITHIN UNDERTHRUSTING LITHOSPHERE AT CONTINENTAL PLATE BOUNDARY Depth of focus: 60 km. Travel times across and away from underthrusting lithosphere. Separate arrival times beyond 65 km caused by trapped critical reflections in the low velocity layer. See Figure 14 for ray paths and velocity distribution.

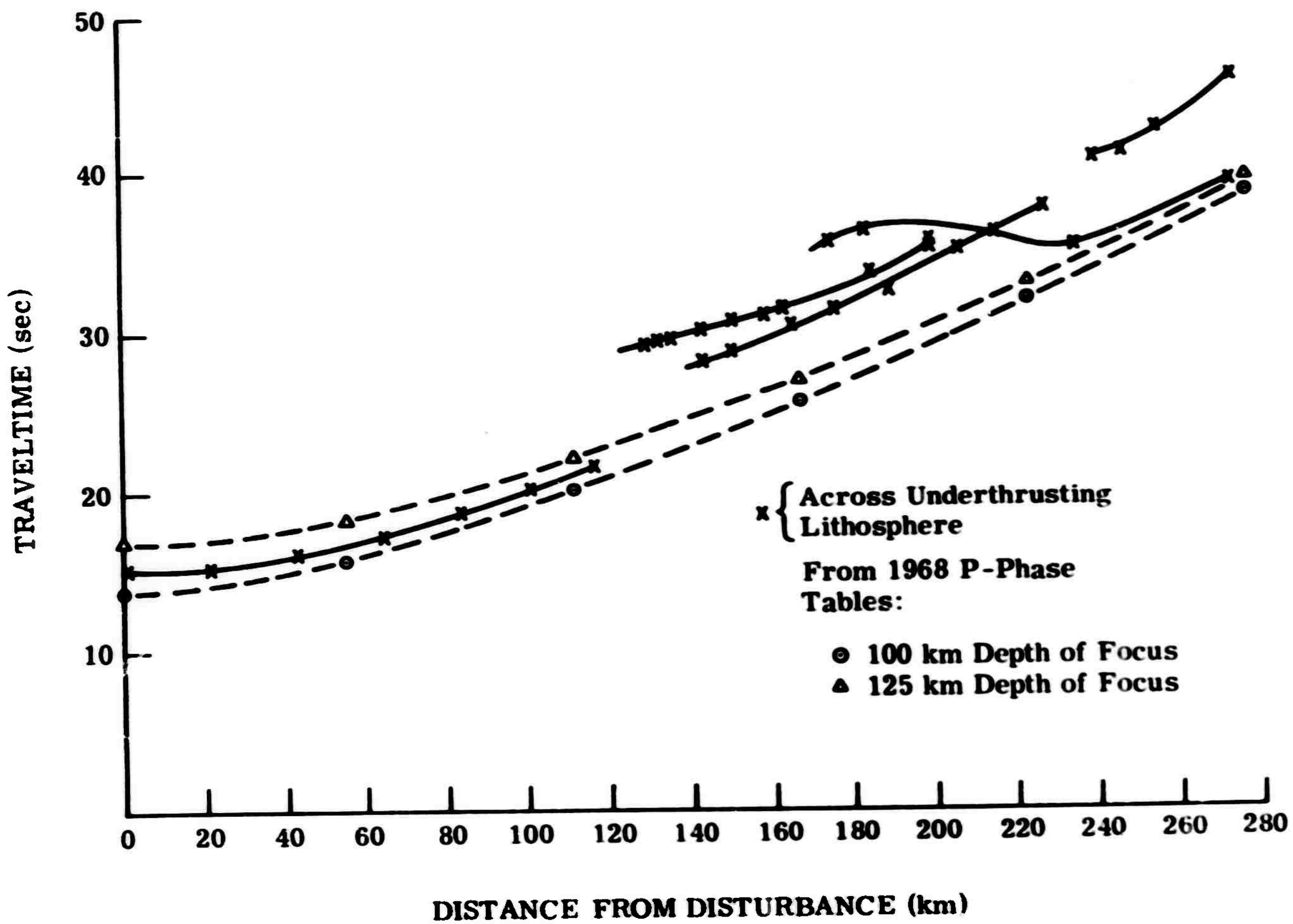

FIGURE 19. EARTHQUAKE WITHIN UNDERTHRUSTING LITHOSPHERE AT CONTINENTAL PLATE BOUNDARY. Depth of focus: 110 km. Travel times across underthrusting lithosphere. Travel times for 100 km and 125 km depth of focus from 1968 P-phase tables shown for comparison. Different arrival times beyond 135 km caused by trapped critical reflections in the low velocity layer. See Figure 15 for ray paths and velocity distribution.

APPENDIX

Listings of Computer Programs

```
C        PROGRAM FOR A FLAT EARTH CROSS-SECTION MODEL
C        MODEL OF VELOCITY DISTRIBUTION IS SET UP AND RAYS ARE PROPAGATED
C        THROUGH THE MODEL.
C        TRAVEL TIMES, DISTANCES, APPROXIMATE AMPLITUDES ARE COMPUTED.
C                  NAMELIST PARAMETERS ALPHABETICALLY LISTED
C
C        A         ARRAY OF SAMPLED VELOCITY VALUES
C        ADIF      MINIMUM ANGLE AT WHICH PRECISE INTERFACE LOCATION IS USED
C        B         ARRAY CONTAINING SAMPLED SLOPE VALUES
C        BEGIN     INITIAL ANGLE OF RAY
C        DEIN      INCREMENTATION ANGLE OF INCIDENT RAYS
C        DEC       EXPONENTIAL DECREMENT BY ATTENUATION
C        DELH      HORIZONTAL INCREMENTATION OF INCIDENT RAYS
C        DELV      VERTICAL INCREMENTATION OF INCIDENT RAYS
C        DIFRAC    DISTANCE IN KM BETWEEN SAMPLE POINTS
C        G         RAY SEGMENT LENGTH
C        ICYCLE    MAXIMUM NUMBER OF RAY INCREMENTATIONS
C        IEXACT    SWITCH TO INCLUDE PRECISE INTERFACE LOCATIONS
C        IMOVIE    SWITCH TO MULTIPLY G BY VELOCITY TO GIVE APPEARANCE OF RAYS
C           PROPAGATING PROPORTIONAL TO VELOCITY WHEN MAKING A COMPUTER MOV
C        INAME     SWITCH TO INCLUDE NAMELIST INPUT OF VELOCITY AND SLOPE DATA
C        IPLANE    SWITCH TO INCLUDE CALCULATIONS FOR INCIDENT PLANE WAVE
C        IPLOT     SWITCH TO INCLUDE CALCOMP PLOTTING
C        IPORSH    LIMIT OF NUMBER OF HORIZONTAL SAMPLE POINTS
C        JPORSH    LIMIT OF VERTICAL NUMBER OF VERTICAL SAMPLE POINTS
C        IWRITE    SWITCH TO INCLUDE DIAGNOSTIC WRITEOUTS
C        MATH      SWITCH TO INCLUDE CONTINUOUS MATHEMATICAL FUNCTION
C        NMULT     NUMBER OF MULTIPLE REFLECTIONS ALLOWED
C        NOB       NUMBER OF INITIAL ANGLE INCREMENTS
C        NOH       NUMBER OF INITIAL HORIZONTAL INCREMENTS
C        NOIN      TOTAL NUMBER OF INITIAL RAYS
C        NOV       NUMBER OF INITIAL VERTICAL INCREMENTS
C        NULINE    NUMBER OF CALCOMP LINES NECESSARY TO OUTLINE STRUCTURE
C        ORIGH     INITIAL HORIZONTAL POSITION OF RAY
C        ORIGV     INITIAL VERTICAL POSITION OF RAY
C        PNM       INITIAL AMPLITUDE VALUE
C        REFVEL    REFERENCE VELOCITY TO MAKE MOVIE
C        SCA       PLOT SCALING FACTOR
C        TEST      SMALL VALUE TO TEST REPEATED EMERGENT ANGLE DIFFERENCES
C        VELDIF    VELOCITY DIFFERENCE NECESSARY TO CAUSE REFLECTION.
C        X         ARRAY CONTAINING HORIZONTAL POSITIONS FOR PLOTTING
C        Y         ARRAY CONTAINING VERTICAL POSITIONS FOR PLOTTING
C
C                   SYMBOL TABLE
C        AMPL      AMPLITUDE
C        ANGLE1    INCIDENT ANGLE OF RAY
C        ANGLE2    EMERGING ANGLE OF RAY
C        ANT       ARRAY FOR AMPLITUDES AFTER REFLECTIONS
C        AT        ARRAY FOR TRAVEL TIMES
C        CLJ       ARRAY FOR VALUES OF ANGLES TO BE RETURNED TO AFTER REFLECTI
C        COMBEG    SUBROUTINE TO INCREMENT INITIAL ANGLE OF INCIDENCE
C        COMHOR    SUBROUTINE TO INCREMENT INITIAL HORIZONTAL POSITION
C        COMVER    SUBROUTINE TO INCREMENT INITIAL VERTICAL POSITION
C        DISTA     DISTANCE FROM SOURCE TO EMERGENCE OF RAY
C        DKM       DISTANCE FROM EDGE OF FIELD TO EMERGENCE OF RAY
C        FFF       SINE OF EMERGING ANGLE(ABOVE 1.DO IF CRITICAL ANGLE)
C        GINBAD    SUBROUTINE TO BRING RAY END PRECISELY TO INTERFACE
C        GG        VERTICAL INCREMENT FOR PROBING ANGLE
C        GGG       VERTICAL INCREMENT
C        HH        HORIZONTAL INCREMENT FOR EMERGING ANGLE
```

```
C        HHH     HORIZONTAL INCREMENT
C        ICRIT   SWITCH TO INDICATE CRITICAL REFLECTION
C        III     INTEGER VALUE OF PIP
C        IINT    SWITCH TO INDICATE RETURN FROM REFLECTED RAY
C        IM      HORIZONTAL PROBED INDEX POSITION
C        INAMI   SWITCH TO INDICATE USE OF SECOND NAMELIST
C        INT     ARRAY FOR VELOCITY VALUES AFTER REFLECTION
C        INTFAC  SWITCH TO INDICATE INTERFACE
C        INUM    NUMBER OF TIMES THROUGH INNER LOOP
C        IS      VERTICAL PROBED INDEX POSITION
C        ISEC    TRAVEL TIME IN INTEGER NUMBER OF MINUTES
C        ISURF   SWITCH TO INDICATE EMERGENCE TO SURFACE
C        IVW     SWITCH TO INDICATE REFLECTION
C        IZ      INDEX FOR NUMBER OF RAY REFLECTION
C        JJJ     INTEGER VALUE OF PIJ
C        NPLOT   NUMBER OF PLOTTING INCREMENTS
C        ON      ARRAY FOR DISTANCES RAYS HAVE TRAVELLED
C        PI      ARRAY FOR VALUES OF HORIZONTAL POSITION
C        PIP     HORIZONTAL POSITION OF RAY SEGEMNT TO DETERMINE VELOCITY
C        PIPROB  HORIZONTAL "PROBED" POSITION
C        PIJ     VERTICAL POSITION OF RAY SEGMENT TO DETERMINE VELOCITY
C        PIT     CONSTANT: 3.14159
C        PITEST  PROBED HORIZONTAL POSITION FOR COMPUTING VELOCITY
C        PITT    CONSTANT: 1.5708
C        PJ      ARRAY FOR VALUES OF VERTICAL POSITION
C        PJPROB  VERTICAL PROBED POSITION
C        PJTEST  PROBED VERTICAL POSITION FOR COMPUTING VELUCITY
C        PLANTT  SUBROUTINE FOR INCIDENT TRAVEL TIME FOR PLANE WAVE
C        POLY    CUBIC FUNCTION TO COMPUTE VELOCITY
C        RIGH    ORIGINAL ORIGH
C        SEC     TRAVEL TIME IN SECONDS ABOVE INTEGER MINUTES
C        SINDET  SUBROUTINE TO COMPUTE EMERGING ANGLE FOR ANY GIVEN ANGLE
C           AND SLOPE
C        SLOP    INTERMEDIATE (SAVED) VALUE OF SLOPE
C        SPEED   PROBED VALUE OF VELOCITY
C        TMIN    TRAVEL TIME IN MINUTES
C        TRENCH  SUBROUTINE TO COMPUTE VELOCITIES FOR UNDERTHRUSTING LITHOSP
C        TSPEED  TENTATIVE VALUE OF VELOCITY
C        VEL     VELOCITY
C
         DIMENSION PI(10), PJ(10), CIJ(10), ON(10), AT(10), ANT(10), INT(10
        1)
         DIMENSION A(121,75), B(121,75), SLPRAT(121,75)
C             ***PLOTTING INTRUCTION***
         DIMENSION X(1000), Y(1000)
C             ***                          ***
         DATA PIT,ICRIT,PITT,INUM,ISURF/3.1415927,0,1.5708,0,0/
         NAMELIST /NAM1/ NOIN,DELN,BEGIN,NMULT,ORIGV,ORIGH,G,IPORSH,JPORSH,
        1DIFRAC,VELDIF,IWRITE,DEC,INAML,NOB,NOV,NOH,DELV,DELH,TEST,MATH,ICY
        2CLE,REFVEL,IMOVIE,IEXACT,PNM,ADIF,X,Y,NULINE,SCA,IPLANE,IPLOT/NAM2
        3/A,B,SLPRAT
         READ (4,NAM1)
         RIGH=ORIGH
         IF (IPLOT.EQ.0) GO TO 1
         CALL PLTOFS (0.,SCA,0.,SCA,4.,3.)
1        IF (INAMI.EQ.1) GO TO 2
         READ (6,34) ((A(I,J),I=1,IPORSH),J=1,JPORSH)
         READ (6,34) ((B(I,J),I=1,IPORSH),J=1,JPORSH)
2        READ (4,NAM2)
         WRITE (7,37)
```

```
      WRITE (7,36)
C             ***PLOTTING INSTRUCTION***
      IF (IPLOT.EQ.0) GO TO 3
      CALL PLINE (X(1),Y(1),NOLINE,1,0,0,1)
 3    CONTINUE
C             ***                              ***
      DO 32 I=1,NOIN
      NPLT=0
      WRITE (7,35) BEGIN,ORIGH,ORIGV
      ANGLE2=BEGIN
      DO 4 I=1,10
      AT(I)=0.
 4    ON(I)=0.
      IF (IPLANE.EQ.1) CALL PLANIT (BEGIN,ORIGH,AT(1),ON(1),RIGH,DIFRAC,
     1REFVEL)
      IVM=0
      IZ=1
      ANT(IZ)=1.
      PI(IZ)=ORIGH
      PJ(IZ)=ORIGV
      I=PI(IZ)+.5001
      J=PJ(IZ)+.5001
      VEL=A(I,J)
      INTFAC=0
      IINT=0
 5    IF (IZ.LE.0) GO TO 31
      NPLT=NPLT+1
      INUM=INUM+1
      IF (INUM.GE.ICYCLE) CALL SYSTEM
      ANGLE1=ANGLE2
C
C
      IF (ANGLE1.GT.7.) GO TO 14
      IF (IMOVIE.EQ.1) G=A(I,J)/REFVEL
C             ***PLOTTING INSTRUCTION***
      IF (IPLOT.EQ.0) GO TO 6
      IF (NPLT.NE.1) GO TO 7
      X(NPLT)=PJ(IZ)
      Y(NPLT)=PI(IZ)
      NPLT=NPLT+1
 6    CONTINUE
C             ***                              ***
 7    HHH=G*SIN(ANGLE1)/2.
      GGG=G*COS(ANGLE1)/2.
      PIP=PI(IZ)+HHH
      PIJ=PJ(IZ)+GGG
      PI(IZ)=PIP+HHH
      PJ(IZ)=PIJ+GGG
      AT(IZ)=AT(IZ)+DIFRAC*G/VEL
      ON(IZ)=ON(IZ)+DIFRAC*G
C             ***PLOTTING INSTRUCTION***
      IF (IPLOT.EQ.0) GO TO 8
      IF (IVEL.EQ.1.AND.NPLT.GE.4) NPLT=NPLT-1
      X(NPLT)=PJ(IZ)
      Y(NPLT)=PI(IZ)
 8    CONTINUE
C             ***                              ***
      IF (PJ(IZ).GE..5) GO TO 11
 9    ISURF=1
      IF (IEXACT.EQ.1) CALL GINMAD (ANGLE1,SLOP,PI(IZ),PJ(IZ),IM,IS,AT(I
```

```
     1Z).ON(IZ).DIFRAC.VEL,ISURF.ADIF.SLPRAT(I,J))
      ISURF=0
      DISTA=(PI(IZ)-ORIGH)*DIFRAC
      DKM=(PI(IZ)-1.)*DIFRAC
      AMPL=(PUM*ANT(IZ)/ON(IZ))*EXP(DEC*ON(IZ))
      TMIN=AT(IZ)/60.
      ISEC=TMIN
      SEC=60.*(TMIN-ISEC)
      WRITE (7,35) DISTA,DKM,TMIN,SEC,AT(IZ),AMPL
      IF (IWRITE.EQ.1) WRITE (7,35) PJ(IZ),PI(IZ),ANGLE1
C          ***PLOTTING INSTRUCTION***
      IF (IPLOT.EQ.0) GO TO 10
      X(NPLT)=PJ(IZ)
      Y(NPLT)=PI(IZ)
      NPLT=NPLT-1
      CALL PLINE (X(2),Y(2),NPLT,1,0,0,1)
      NPLT=0
10    CONTINUE
C          ***                              ***
      GO TO 14
11    IF (PI(IZ).GT..5.AND.PJ(IZ).GT..5.AND.PJ(IZ).LT.JPORSH-.5.AND.PI(I
     1Z).LT.IPORSH-.5) GO TO 15
C           ***  PLOTTING INSTRUCTION   ***
      IF (IPLOT.EQ.0) GO TO 12
      NPLT=NPLT-1
      CALL PLINE (X(2),Y(2),NPLT,1,0,0,1)
      NPLT=0
12    CONTINUE
C           ***                              ***
13    CONTINUE
      WRITE (7,38) PI(IZ),PJ(IZ),ANGLE1,ON(IZ),AT(IZ)
C     PI(IZ)=PI(IZ)+1.-ORIGH
C     CALL WORLD(PI,PJ,ON,ANT,AT,ANGLE2,IZ,X,Y,LI)
14    IF (IZ.EQ.1) GO TO 31
      IVW=0
      ANT(IZ-1)=(1-ANT(IZ))*ANT(IZ-1)
      IZ=IZ-1
      I=PIP+.5
      J=PIJ+.5
      ANGLE2=CIJ(IZ)
      GO TO 5
15    I=PI(IZ)+.5
      J=PJ(IZ)+.5
C
      PIPROB=PIP+HHH
      PJPROB=PIJ+GGG
      IM=PIPROB+.5
      IS=PJPROB+.5
      IF (IWRITE.EQ.0) GO TO 16
      WRITE (1,35) PJ(IZ),PI(IZ),VEL,SLOP,ANGLE1
16    IF (INTEAC.EQ.1) GO TO 18
      IF (MATH.EQ.1) GO TO 17
      IF (IS.LT.1) GO TO 9
      IF (IM.GT.IPORSH.OR.IS.GT.JPORSH.OR.IM.LT.1) GO TO 13
      III=PIP+.5
      JJJ=PIJ+.5
      VEL=A(III,JJJ)+GG*(PIJ-JJJ)*(A(III,JJJ-1)-A(III,JJJ))
      GO TO 18
17    CALL TRENCH (PIP,PIJ,VEL,SLOPE,DIFRAC)
      IF (IINT.EQ.1) VEL=INT(IZ)
```

```
 18     IVEL=0
        IF (INTFAC.EQ.1) SLOPE=SLOP
        INTFAC=0
        IINT=0
        IF (IVW.NE.1) GO TO 19
        IVW=0
        GO TO 5
 19     TSPEED=VEL
        ICRIT=0
        DO 24 IJKL=1,10
        HH=G*SIN(ANGLE2)
        GG=G*COS(ANGLE2)
        PITEST=PI(IZ)+HH/2.
        PJTEST=PJ(IZ)+GG/2.
        IF (PJTEST.LT.1.) GO TO 9
        IF (MATH.EQ.1) GO TO 20
        III=PITEST+.5
        JJJ=PJTEST+.5
        SPEED=A(III,JJJ)+GG*(PJTEST-JJJ)*(A(III,JJJ-1)-A(III,JJJ))
        SLOP=B(I,J)
        IF (SPEED.EQ.VEL.AND.IJKL.EQ.1) GO TO 22
        IF (ANGLE1.GT.PITT+.5585.AND.ANGLE1.LT.3.*PITT+.5585) SLOP=B(IM,IS
       1)
        GO TO 21
 20     CALL TRENCH (PITEST,PJTEST,SPEED,SLP,DIFRAC)
        IF (SPEED.EQ.VEL.AND.IJKL.EQ.1) GO TO 22
 21     IVW=0
        SLOP=SLP
        GO TO 23
 22     IVEL=1
        GO TO 5
 23     CONTINUE
        IF (ANGLE1.GT.PITT+.5585.AND.ANGLE1.LT.3.*PITT+.5585.AND.ABS(VEL-S
       1PEED).GT..7) SLOP=SLOPE
        IF (ABS(SPEED-TSPEED).LT.TEST) GO TO 25
        TSPEED=SPEED
        CALL SINDET (ANGLE1,ANGLE2,VEL,SPEED,SLOP,ICRIT,FFF)
 24     CONTINUE
 25     CONTINUE
        IF (IWRITE.EQ.1) WRITE (1,35) VEL,SPEED,ANGLE2,ANGLE1,SLOP,PJTEST
        IF (ABS(SPEED-VEL).GE.VELDIF.AND.IZ.LE.NMULT.AND.IVW.LT.1.OR.ICRIT
       1.EQ.1) GO TO 26
        VEL=SPEED
        INTFAC=1
        IVW=0
        GO TO 5
 26     IF (IWRITE.EQ.1) WRITE (1,33) PI(IZ),PJ(IZ),IM,IS
        IF (IEXACT.LT.2) GO TO 27
        CALL GINMAD (ANGLE1,SLOP,PI(IZ),PJ(IZ),IM,IS,AT(IZ),ON(IZ),DIFRAC,
       1VEL,ISURF,ADIF,SLPRAT(I,J))
 27     CONTINUE
        IF (IWRITE.EQ.1) WRITE (1,33) PI(IZ),PJ(IZ),IM,IS
C            ***PLOTTING INSTRUCTION***
        IF (IPLOT.EQ.0) GO TO 28
        X(NPLT)=PJ(IZ)
        Y(NPLT)=PI(IZ)
        NPLT=NPLT-1
        CALL PLINE (X(2),Y(2),NPLT,1,0,0,1)
        NPLT=0
 28     CONTINUE
```

```
C               ***                                ***
        IF (ICRIT+IVW.GE.2) GO TO 5
        IF (ICRIT.EQ.1) GO TO 30
        IVW=1
        IZ=IZ+1
        ON(IZ)=ON(IZ-1)
        AT(IZ)=AT(IZ-1)
        PI(IZ)=PI(IZ-1)
        PJ(IZ)=PJ(IZ-1)
        ANT(IZ)=ANT(IZ-1)*(SPEED-VEL)/(SPEED+VEL)
        VEL=SPEED
        CIJ(IZ-1)=ANGLE2
        INT(IZ-1)=SPEED
        IF (IZ-NMULT) 30,29,29
29      IZ=IZ-1
        IINT=1
        GO TO 5
30      ANGLE2=PIT-ANGLE1+2.*SLOP
        IF (ANGLE2.GE.2.*PIT) ANGLE2=ANGLE2-2.*PIT
        IF (ANGLE2.LT.0.) ANGLE2=ANGLE2+2.*PIT
        IVW=1
        GO TO 5
31      CONTINUE
        CALL COMBEG (BEGIN,DELN,NOB,IB)
        CALL COMVER (ORIGV,DELV,NOV,IV)
        CALL COMHOR (ORIGH,DELH,NOH,IH)
32      CONTINUE
C               ***PLOTTING INSTRUCTION***
        CALL PLTEND
C
C
33      FORMAT ('0',2F6.2,2I4)
34      FORMAT (10F4.2)
35      FORMAT (''6F10.2)
36      FORMAT ('DISTANCEKILOMETERSTT(MIN)TT(SEC)','TOTAL(SEC)AMPLITUDE')
37      FORMAT ('0INITIALANGLE,INITIALDISTANCE,INITIALDEPTH')
38      FORMAT ('0NON-SURFACE:',5F7.3)
        END
C
        SUBROUTINE GINMAD (ANGLE1,B,PI,PJ,IM,IS,AT,ON,D,A,ISURF,ADIF,SLPRA
       1T)
        DATA PIE2,PIE,PIE32/1.57079,3.1459,4.712397/
        ANDIF1=ABS(ANGLE1-B-PIE2)
        ANDIF2=ABS(ANGLE1-B-PIE32)
        IF (ANDIF1.LT.ADIF.OR.ANDIF2.LT.ADIF) RETURN
        IF (ISURF.EQ.0) GO TO 1
        DV=1.-PJ
        ANG=ABS(COS(PIE-ANGLE1))
        IF (ANG.LT..02) ANG=.02
        GIN=-DV/ANG
        GO TO 7
1       V=SLPRAT/100.
        IH=SLPRAT
        H=SLPRAT-IH
        IF (V.LT..01) V=.5
        IF (H.LT..01) H=.5
        DV=IS-PJ-1.+V
        DH=IM-PI-1.+H
        IF (ANGLE1.GT.PIE2.AND.ANGLE1.LT.PIE32) DV=DV+1.
        IF (ANGLE1.GT.PIE) DH=DH+1.
```

```
      IF (ABS(DH).LT..001) DH=.001
      DVH=DV/DH
      H=SQRT(DV*DV+DH*DH)
      GAMMA=ATAN(DVH)
      IF (DH*DV.GT.0.) GO TO 2
      DELT=ABS(GAMMA)-B
      GO TO 3
    2 CONTINUE
      DELT=ABS(GAMMA)+B
    3 CONTINUE
      GIN=ABS(H*SIN(DELT)/SIN(ANGLE1))
      BB=B
      IF (B.GT.1.5) BB=1.5
      IF (B.LT.-1.5) BB=-1.5
      IF (ANGLE1.LT.PIE2) GO TO 6
      IF (ANGLE1.GE.PIE2.AND.ANGLE1.LT.PIE) GO TO 5
      IF (ANGLE1.GE.PIE.AND.ANGLE1.LT.PIE32) GO TO 4
      IF (DV.LT.-DH*TAN(BB)) GIN=-GIN
      IF (BB.GT.ANGLE1-PIE32) GIN=-GIN
      GO TO 7
    4 CONTINUE
      IF (DV.GT.-DH*TAN(BB)) GIN=-GIN
      IF (BB.LT.0..AND.ABS(BB).GT.PIE32-ANGLE1) GIN=-GIN
      GO TO 7
    5 CONTINUE
      IF (DV.GT.-DH*TAN(BB)) GIN=-GIN
      IF (BB.GT.ANGLE1-PIE2) GIN=-GIN
      GO TO 7
    6 CONTINUE
      IF (DV.LT.-DH*TAN(BB)) GIN=-GIN
      IF (BB.LT.0..AND.ABS(BB).GT.PIE2-ANGLE1) GIN=-GIN
    7 CONTINUE
      PI=GIN*SIN(ANGLE1)+PI
      PJ=GIN*COS(ANGLE1)+PJ
      AT=AT+D*GIN/A
      DN=DN+D*GIN
      RETURN
      END
C
      SUBROUTINE TRENCH (PIP,PIJ,VEL,SLOPE,DIFRAC)
    1 IF (PIJ.GE.3.2) GO TO 2
      VEL=6.
      SLOPE=0.
      GO TO 12
    2 IF (PIP.LT.60.2) GO TO 4
      IF (PIJ.GE.7.2) GO TO 3
      VEL=6.75
      SLOPE=0.
      GO TO 12
    3 DEPTH=40.+(PIJ-7.2)*DIFRAC
      VEL=POLY(DEPTH)
      SLOPE=0.
      GO TO 12
    4 IF (PIJ.GT.5.) GO TO 5
      VEL=6.
      SLOPE=0.
      GO TO 6
    5 VEL=6.75
      SLOPE=0.
    6 GRADE1=.848*(PIJ-3.8)+.529*(PIP-50.5)
```

```
      IF (GRADE1.GT.0.) GO TO 8
      IF (PIJ.GT.9.) GO TO 7
      GO TO 12
7     DEPTH=40.+(PIJ-9.)*DIFRAC
      VEL=POLY(DEPTH)
      SLOPE=0.
      GO TO 12
8     IF (1.7724*(PIP-45.7).GT.PIJ-6.8) GO TO 10
      IF (GRADE1.GT.4.) GO TO 9
      VEL=6.75
      SLOPE=.5585
      GO TO 12
9     DEPTH=40.+(.848*(PIJ-8.7)+.529*(PIP-50.5))*DIFRAC
      VEL=POLY(DEPTH)
      SLOPE=.5585
      GO TO 12
10    DENOM=60.2-PIP
      IF (DENOM.LT..001) DENOM=.001
      SLOPE=1.5708-ATAN((32.5-PIJ)/DENOM)
      RAD=SQRT((32.5-PIJ)**2.+DENOM**2.)
      IF (RAD.GT.29.3) GO TO 12
      IF (RAD.GT.25.3) GO TO 11
      DEPTH=40.+(25.3-RAD)*DIFRAC
      VEL=POLY(DEPTH)
      GO TO 12
11    VEL=6.75
      SLOPE=0.
12    RETURN
      END
C
      FUNCTION POLY (X)
      POLY=7.4639+.29893E-02*X-.25167E-04*X*X+.1057E-06*X*X*X
      RETURN
      END
C
      SUBROUTINE SINDET (CIU,CIJ,AIJ,AMS,BIJ,ICRIT)
      ICRIT=0
      DATA PI/3.1415927/
      DATA PPI/1.5707963/
      AA=AMS/AIJ
      GH=0.
      HG=1.
      CC=CIU-BIJ
      IF (CC.EQ.0..OR.CC.EQ.PI.OR.CC.EQ.2.*PI) GO TO 5
      IF (CIU.GE.0..AND.CIU.LE.PI) GO TO 1
      IF (CC.LT.3.*PPI) GO TO 2
      GH=2.*PI
      GO TO 3
1     IF (CC.GT.PPI) GO TO 2
      GO TO 3
2     GH=PI
      HG=-1.
3     FFF=AA*SIN(CC)
      IF (ABS(FFF).GE.1.) GO TO 4
      ARC=HG*ARSIN(FFF)
      CIJ=ARC+BIJ+GH
      IF (CIJ.GT.2.*PI) CIJ=CIJ-2.*PI
      IF (CIJ.LT.0.) CIJ=CIJ+2.*PI
      GO TO 6
4     ICRIT=1
```

```
      GO TO 6                                                    F  26
5     CTJ=CTO                                                    F  27
6     RETURN                                                     F  28
      END                                                        F  29*
C
      SUBROUTINE COMREG (R,DR,N,IR)                              F   1
      IF (IR.GT.N) RETURN                                        F   2
      IR=IR+1                                                    F   3
      R=R+DR                                                     F   4
      RETURN                                                     F   5
      END                                                        F   6*
C
      SUBROUTINE CONVER (V,DV,N,IV)                              G   1
      IF (IV.GT.N) RETURN                                        G   2
      IV=IV+1                                                    G   3
      V=V+DV                                                     G   4
      RETURN                                                     G   5
      END                                                        G   6*
C
      SUBROUTINE COMHOR (H,DH,N,IH)                              H   1
      IF (IH.GT.N) RETURN                                        H   2
      IH=IH+1                                                    H   3
      H=H+DH                                                     H   4
      RETURN                                                     H   5
      END                                                        H   6*
C
      SUBROUTINE PLANIT (R,ORIG,AT,DN,R,DIF,REF)                 I   1
      DN=(R-ORIG)*SIN(R)*DIF                                     I   2
      AT=DN/REF                                                  I   3
      RETURN                                                     I   4
      END                                                        I   5*
                                                   953 LINES PRINTED
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```
C     PROGRAM FOR A SPHERICAL EARTH CROSS-SECTION
C     MODEL OF VELOCITY DISTRIBUTION IS SET UP AND RAYS ARE PROPAGATED
C     THROUGH THE MODEL
C     TRAVEL TIMES, DISTANCES IN DELTA DEGREES AND KILOMETERS, AND
C     APPROXIMATE AMPLITUDES ARE COMPUTED.
C
C                NAMELIST PARAMETERS ALPHABETICALLY LISTED
C     CIRC1   SIZE OF CIRCLE TO BE PLOTTED FOR EARTH'S CIRCUMFERENCE
C     CIRC2   SIZE OF CIRCLE TO BE PLOTTED FOR CORE-MANTLE BOUNDARY
C     CIRC3   SIZE OF CIRCLE TO BE PLOTTED FOR INNER CORE
C     CMAX    MAXIMUM RADIAL ANGLE TO WHICH CIRCLES ARE PLOTTED
C     CMIN    MINIMUM RADIAL ANGLE TO WHICH CIRCLES ARE PLOTTED
C     DECLOG  EXPONENTIAL DECREMENT BY ATTENUATION
C     DEGKM   KILOMETERS PER DEGREE OF EARTH'S ARC
C     EDGE    SMALL DISTANCE TO WHICH INTERFACE SHOULD BE APPROACHED
C     FACDAT  FACTOR TO MULTIPLY DATA TO CHANGE VELOCITY INPUT
C     HORMAX  MAXIMUM HORIZONTAL POSITION
C     HORMIN  MINIMUM HORIZONTAL POSITION
C     IAAP    SWITCH TO INCLUDE APPARENT VELOCITY
C     ICIRC   SWITCH TO INDICATE PLOTTING OF CIRCLES
C     ICYCLE  MAXIMUM NUMBER OF RAY INCREMENTATIONS
C     IDAT    SWITCH TO CHANGE PORTION OF DATA BY FACTOR FACDAT
C     IDIFUS  50% REFLECTION OF AMPLITUDE (DIFFUSION COEFFICIENT)
C     IPLOT   SWITCH TO INCLUDE CALCOMP PLOTTING
C     IPP     SWITCH TO TRACE MULTIPLE P-RAYS (PP,PPP,ETC.)
C     ISAMP   NUMBER OF SAMPLE POINTS TO CENTER OF EARTH
C     IWRITE  SWITCH TO INCLUDE DIAGNOSTIC WRITEOUTS
C     MAXDAT  MAXIMUM NUMBER IN DATA LIST TO BE CHANGED BY FACDAT
C     MAXDEG  MAXIMUM DELTA AT WHICH OUTPUT IS WISHED
C     MAXDKM  MAXIMUM DISTANCE TO WHICH OUTPUT IS WISHED
C     MINDAT  MINIMUM NUMBER IN DATA LIST TO BE CHANGED BY FACDAT
C     MINDEG  MINIMUM DELTA AT WHICH OUTPUT IS WISHED
C     MINDKM  MINIMUM DISTANCE TO WHICH OUTPUT IS WISHED
C     NUMMLT  NUMBER OF MULTIPLE REFLECTIONS ALLOWED
C     PNM     INITIAL AMPLITUDE VALUE
C     PRECI   LOCATION OF INTERFACE BETWEEN SAMPLE POINTS
C     LIMHOR  NUMBER OF HORIZONTAL SAMPLE POINTS USED
C     LIMINC  NUMBER OF TIMES INTERFACE OR SURFACE SHOULD BE APPROACHED
C     LIMVER  NUMBER OF VERTICAL SAMPLE POINTS USED
C     SAMPKM  NUMBER OF KILOMETERS BETWEEN SAMPLE POINTS
C     SEGMEN  RAY SEGMENT LENGTH
C     SURF    MINIMUM HORIZONTAL POSITION ALLOWED
C     TSAMP   FLOATING POINT REPRESENTED DISTANCE TO CENTER OF EARTH
C     VELDIF  VELOCITY DIFFERENCE NECESSARY TO CAUSE REFLECTION.
C     VERMAX  MAXIMUM VERTICAL POSITION
C     VERMIN  MINIMUM VERTICAL POSITION
C
C               SYMBOL TABLE
C     AMPL    AMPLITUDE
C     ANGLE1  INCIDENT ANGLE OF RAY
C     ANGLE2  EMERGING ANGLE OF RAY
C     ANT     ARRAY FOR AMPLITUDES AFTER REFLECTIONS
C     AT      ARRAY FOR TRAVEL TIMES
C     APPVEL  APPARENT VELOCITY (DISTANCE IN KM DIVIDED BY TRAVEL TIME)
C     ARG     VERTICAL POSITION DIVIDED BY HORIZONTAL POSITION
C     BORD    SUBROUTINE TO PRECISELY END RAY AT SURFACE
C     CLI     ARRAY FOR VALUES OF ANGLES TO BE RETURNED TO AFTER REFLECTI
C     D       RADIAL VELOCITY DISTRIBUTION OF A SPHERICAL EARTH
C     DEPTH   MAXIMUM DEPTH REACHED BY RAY (6371 KM-RADIUS)
C     DKM     DISTANCE IN KM
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```
C       DISTA  DELTA IN DEGREES
C       EDGES  SUMMATION OF SMALL INCREMENTS WHEN APPROACHING INTERFACE
C       EEE    SINE OF EMERGING ANGLE(ABOVE 1.00 IF CRITICAL ANGLE)
C       HORINC HORIZONTAL INCREMENTATION OF RAY SEGMENT
C       HORZKM HORIZONTAL POSITION IN KILOMETERS
C       ICRIT  SWITCH TO INDICATE CRITICAL REFLECTION
C       IINTER SWITCH FOR HAVING REACHED INTERFACE
C       IMERGE SWITCH FOR HAVING EMERGED AT SURFACE WITH RAY
C       INUM   NUMBER OF TIMES THROUGH INNER LOOP
C       ISEC   TRAVEL TIME IN INTEGER NUMBER OF MINUTES
C       IVELOC SWITCH TO INDICATE REFLECTION AT INTERFACE
C       IVR    SWITCH FOR REACHING REFLECTION (CRITICAL OR NON-CRITICAL)
C       IVRR   COUNT FOR NUMBER OF SEQUENTIAL CRITICAL REFLECTIONS
C       IZ     INDEX FOR NUMBER OF RAY REFLECTION
C       JCRIT  SWITCH FOR HAVING REACHED CRITICAL ANGLE
C       KSPEED INTEGER VALUE OF DEPTH OF PROBED VALUE
C       KVEL   VERTICAL POSITION IN INTEGER NUMBER OF SAMPLE POINTS
C       KVEL1  VERTICAL POSITION IN NEAREST INTEGER NUMBER OF SAMPLES
C       L      SWITCH TO READ DATA ON FIRST CALL WHEN USED AS SUBROUTINE
C       NUMTIP TOTAL NUMBER OF REFLECTIONS
C       NPLOT  NUMBER OF PLOTTING INCREMENTS
C       ON     ARRAY FOR DISTANCES RAYS HAVE TRAVELLED
C       PI     ARRAY FOR VALUES OF HORIZONTAL POSITION
C       PI1    HORIZONTAL POSITION IN SAMPLE POINTS FROM REFERENCE
C       PIT    CONSTANT: 3.14159
C       PI11   PROBED VALUE OF HORIZONTAL POSITION FROM REFERENCE
C       PI12   HORIZONTAL POSITION IN SAMPLE POINTS FROM REFERENCE
C       PIPROB PROBED VALUE OF HORIZONTAL POSITION
C       PIREF  HORIZONTAL POSITION FOR DETERMINATION OF VELOCITY
C       PJ     ARRAY FOR VALUES OF VERTICAL POSITION
C       PJJ    VERTICAL POSITION IN SAMPLE POINTS FROM CENTER OF EARTH
C       PJJ1   PROBED VALUE OF VERTICAL POSITION FROM CENTER OF EARTH
C       PJJ2   VERTICAL POSITION IN SAMPLE POINTS FROM CENTER OF EARTH
C       PJPROB PROBED VALUE OF VERTICAL POSITION
C       PJREF  VERTICAL POSITION FOR DETERMINATION OF VELOCITY
C       RAD1   DISTANCE FROM CENTER OF EARTH IN SAMPLE POINTS
C       RAD3   DISTANCE FROM CENTER OF EARTH
C       RADIUS DISTANCE FROM CENTER OF EARTH IN SAMPLE POINTS
C       RADUS  MINIMUM RADIUS REACHED BY RAY IN KILOMETERS
C       RPOINT SUBROUTINE TO PRECISELY REFLECT RAY FROM INTERFACE
C       SEC    TRAVEL TIME IN SECONDS ABOVE INTEGER MINUTES
C       SINDET SUBROUTINE TO COMPUTE EMERGING ANGLE FOR ANY GIVEN ANGLE
C          AND SLOPE
C       SLOPE  SLOPE
C       SLOPER SAVED VALUE OF SLOPE
C       SPEED  DEPTH OF PROBED LOCATION
C       TMIN   TRAVEL TIME IN MINUTES
C       VEL    VERTICAL DISTANCE FROM CENTER OF EARTH IN SAMPLE POINTS
C       VELO1  VELOCITY
C       VELO1R VELOCITY SAVED UPON REFLECTION
C       VELO2  VELOCITY
C       VELO3  INTERPOLATION OF DEPTH IN SAMPLE POINTS
C       VELO4  VELOCITY
C       VELOCY ARRAY TO HOLD PREVIOUS VELOCITY VALUES AFTER REFLECTIONS
C       VERINC VERTICAL INCREMENTATION OF RAY SEGMENT
C       VERTKM DEPTH IN KILOMETERS
C
        SUBROUTINE WORLD (PI,PJ,ON,ANT,AT,ANGLE2,IZ,X,Y,L)
        DIMENSION PI(10), PJ(10), CIJ(10), ON(10), ANT(10), AT(10), D(1000
       1), VELOCY(10)
```

```
      DIMENSION X(1000), Y(1000)
      DATA PIT,PIT2,ICRIT,INUM,IV,IV,IH/3.14159,1.5708,0,0,0,0,0/
      IF (I.GT.1) GO TO 4
      NAMELIST /DATAIN/ SEGMEN,ISAMP,LIMVER,LIMHOR,SAMPKM,VELDIF,DECLUG,
     1NUMULT,ICYCLE,EDGE,SURF,TSAMP,PNM,LIMINC,IWRITE,PRECI,SCA,CIRC1,CI
     2RC2,CIRC3,HORMIN,HORMAX,VERMIN,VERMAX,DEGKM,IDAT,IPLOT,IPP,MINDAT,
     3MAXDAT,FACDAT,IDIFUS,IAAP,ICIRC,MINDEG,MAXDEG,CMIN,CMAX,MINDKM,MAX
     4DKM
      READ (5,DATAIN)
      READ (5,27) (D(I),I=1,LIMVER)
      IF (IDAT.EQ.0) GO TO 2
      DO 1 I=MINDAT,MAXDAT
      D(I)=D(I)*FACDAT
    1 CONTINUE
    2 CONTINUE
      WRITE (7,29)
C     **************PLOTTING INSTRUCTION********************
      IF (IPLOT.EQ.0) GO TO 4
      CALL PLTOFS (0.,SCA,0.,SCA,4.,6.)
      IF (ICIRC.EQ.0) GO TO 3
      CALL PCIRCL (TSAMP,1.,CMIN,CMAX,CIRC1,CIRC1,0,1)
      CALL PCIRCL (TSAMP,1.,CMIN,CMAX,CIRC3,CIRC3,0,1)
      CALL PCIRCL (TSAMP,1.,CMIN,CMAX,CIRC2,CIRC2,0,1)
    3 CONTINUE
    4 CONTINUE
      VERTKM=(PJ(17)-1.)*SAMPKM
      HORZKM=(PI(17)-1.)*SAMPKM
      NPLT=0
      MULTIP=0
      KVEL=PJ(17)
      IF (KVEL.GT.LIMVER-1) KVEL=2.*SAMPKM-1-KVEL
      KVELT=PJ(17)+.5
      VEL=TSAMP-PJ(17)
      VELD3=D(KVEL)+.5*SEGMEN*(D(KVEL+1)-D(KVEL))*ABS(SIN(ANGLE2))
      VELD2=VELD3
      RADIUS=TSAMP-PJ(17)
      HORINC=SEGMEN*SIN(ANGLE2)
      VERINC=SEGMEN*COS(ANGLE2)
      ICRIT=0
      IINTER=0
      IMERGE=0
      IVW=0
      IVWR=0
      EDGES=0.
    5 CONTINUE
C     **************PLOTTING INSTRUCTION********************
      IF (IPLOT.EQ.0) GO TO 7
      NPLT=NPLT+1
      IF (NPLT.NE.1) GO TO 6
      X(NPLT)=PJ(17)
      Y(NPLT)=PI(17)
      NPLT=NPLT+1
    6 CONTINUE
    7 CONTINUE
C     ******************************************************
      INUM=INUM+1
      IF (INUM.GE.ICYCLE) RETURN
      ANGLE1=ANGLE2
      PIREF=PI(17)+.5*HORINC
      PJREF=PJ(17)+.5*VERINC
```

```
      PI(IZ)=PI(IZ)+HORINC
      PJ(IZ)=PJ(IZ)+VERINC
      IF (PI(IZ).LT.HORMIN.OR.PI(IZ).GT.HORMAX.OR.PJ(IZ).LT.VERMIN.OR.PJ
     1(IZ).GT.VERMAX) GO TO 25
C     ***************PLOTTING INSTRUCTION*****************
      IF (IPLOT.EQ.0) GO TO 8
      X(NPLT)=PJ(IZ)
      Y(NPLT)=PI(IZ)
8     CONTINUE
C     ****************************************************
      IF (IWRITE.GE.1) WRITE (7,28) PI(IZ),PJ(IZ),VELO1,ANGLE1
      IF (PI(IZ).LT.SURF) RETURN
      PII=PIREF-1.
      PJJ=TSAMP-PJREF
      RAD1=SQRT(PII*PII+PJJ*PJJ)
      IF (RADIUS.GT.RAD1) RADIUS=RAD1
      VEL=TSAMP-RAD1
      KVEL=VEL
      KVEL1=VEL+.5
      PII2=ABS(PI(IZ)-1.)
      PJJ2=TSAMP-PJ(IZ)
      IF (PII2.LT..0001) PII2=.0001
      IF (PII.LT.0.) PII2=-PII2
      ARG=PJJ2/PII2
      IF (ARG.GE.0.) GO TO 9
      SLOPE=ATAN(ARG)+PIT2
      GO TO 10
9     CONTINUE
      SLOPE=ATAN(ARG)-PIT2
10    CONTINUE
      IF (IVELOC.EQ.0) GO TO 11
      VELO1=VELO2
      IF (IVW.EQ.1) VELO1=VELO1R
      IVW=0
11    IF (VELO3.LT..05) GO TO 12
      VELO4=VELO3
      AT(IZ)=AT(IZ)+SEGMEN*SAMPKM/VELO4
12    DN(IZ)=DN(IZ)+SEGMEN*SAMPKM
      IF (RAD1.GT.TSAMP-1.) GO TO 23
13    RAD3=RAD1
      ICRIT=0
      IF (IVELOC.EQ.0) VELO1=VELO2
      IVELOC=0
      DO 14 II=1,6
      PIPROB=PI(IZ)+HORINC/2.
      PJPROB=PJ(IZ)+VERINC/2.
      PII1=PIPROB-1.
      PJJ1=TSAMP-PJPROB
      RAD2=SQRT(PII1*PII1+PJJ1*PJJ1)
      IF (RAD2.GT.TSAMP-1.) GO TO 23
      IF (ABS(RAD3-RAD2).LE..003) GO TO 15
      RAD3=RAD2
      SPEED=TSAMP-RAD2
      KSPEED=SPEED
      VELO2=D(KSPEED)+(SPEED-KSPEED)*(D(KSPEED+1)-D(KSPEED))
      IF (ABS(VELO1-VELO2).LT..005) GO TO 5
      CALL SINDET (ANGLE1,ANGLE2,VELO1,VELO2,SLOPE,ICRIT,FFF)
      JCRIT=ICRIT+JCRIT
      IF (JCRIT.GE.3) GO TO 25
      IF (ICRIT.EQ.1) GO TO 16
```

```
      HORINC=SEGMEN*SIN(ANGLE2)
      VERINC=SEGMEN*COS(ANGLE2)
14    CONTINUE
15    CONTINUE
      IF (IVWR.GT.0) IVWR=IVWR-1
      KVEL2=SPEED+.5
      VELO3=VELO2
      IF (IINTER.EQ.1) GO TO 21
      IF (IWRITE.GE.6) WRITE (7,30) VELO1,VELO2,SLOPE,IZ
      IF (IVWR.NE.0) GO TO 17
16    IF (SPEED.GT.VEL.AND.ABS(D(KVEL1)-D(KVEL1+1)).GT.VELDIF) GO TO 19
      IF (VEL.GT.SPEED.AND.ABS(D(KVEL1)-D(KVEL1-1)).GT.VELDIF) GO TO 19
17    CONTINUE
      IF (ICRIT.EQ.0) GO TO 18
      SLOPER=SLOPE
      GO TO 22
18    CONTINUE
      GO TO 5
19    CONTINUE
      VELO3=VELO1
      CALL RPOINT (ANGLE1,SLOPE,PI(IZ),PJ(IZ),TSAMP,EDGE,VEL,ON(IZ),AT(I
     1Z),SAMPKM,VELO3,PIT,PIT2,ARG,SPEED,LIMINC,RADAR,PRECI,EDGES,KVEL1)
C     **************PLOTTING INSTRUCTION*******************
      IF (IPLOT.EQ.0) GO TO 20
      X(NPLT)=PJ(IZ)
      Y(NPLT)=PI(IZ)
      CALL PLINE (X(1),Y(1),NPLT,1,0,0,1)
      NPLT=0
20    CONTINUE
C     ************************************************************
      IF (IWRITE.GE.5) WRITE (7,30) PI(IZ),ANGLE1,SLOPE
      SLOPER=SLOPE
      IF (ICRIT.EQ.1) GO TO 22
      IINTER=1
      VELO2=VELO1
      VELO1R=VELO1
      GO TO 13
21    IINTER=0
      IVELOC=1
      IF (IZ.EQ.NUMULT) GO TO 5
      IZ=IZ+1
      IF (IZ.GT.NUMULT) RETURN
      ON(IZ)=ON(IZ-1)
      AT(IZ)=AT(IZ-1)
      PI(IZ)=PI(IZ-1)
      PJ(IZ)=PJ(IZ-1)
      JCRIT=0
      ANT(IZ)=ANT(IZ-1)*((D(KVEL1)-D(KVEL2))/(D(KVEL1)+D(KVEL2)))
      IF (IDIFUS.EQ.1) ANT(IZ)=.5*ANT(IZ-1)
      VELOCY(IZ-1)=VELO2
      CIJ(IZ-1)=ANGLE2
      IF (RADIUS.GT.RADAR) RADIUS=RADAR
      IVWR=3
22    CONTINUE
      IMERGE=0
      MULTIP=MULTIP+1
      VELO1R=VELO1
      ANGLE2=PIT-ANGLE1+2.*SLOPER
      IF (IWRITE.GE.4) WRITE (7,30) ANGLE1,SLOPER,ANGLE2
      IVELOC=1
```

```
      IF (ANGLE2.GT.2.*PIT) ANGLE2=ANGLE2-2.*PIT
      IF (ANGLE2.LT.0.) ANGLE2=ANGLE2+2.*PIT
      HORINC=(SEGMEN+EDGES)*SIN(ANGLE2)
      VERINC=(SEGMEN+EDGES)*COS(ANGLE2)
      AT(IZ)=AT(IZ)+SAMPKM*EDGES/VELO4
      DN(IZ)=DN(IZ)+EDGES*SAMPKM
      VELO3=VELO4
      EDGES=0.
      IVW=1
      IVWR=3
      GO TO 5
   23 CONTINUE
      IF (IWRITE.GE.3) WRITE (7,28) AT(IZ),DN(IZ),VELO1,SAMPM,ANGLE1,PI(
     1IZ),PJ(IZ)
      CALL BDRD (PI(IZ),PJ(IZ),ANGLE1,ISAMP,EDGE,DN(IZ),AT(IZ),SAMPKM,D(
     1KVEL),DISTA)
      IF (PI(IZ).LT.1.) DISTA=DISTA+180.
      DKM=DEGKM*DISTA
      APPVEL=DKM/AT(IZ)
      AMPL=(PNM*ANT(IZ)/DN(IZ))*EXP(DECLOG*DN(IZ))
      TMIN=AT(IZ)/60.
      ISEC=TMIN
      SEC=60.*(TMIN-ISEC)
      IF (IWRITE.GE.2) WRITE (7,28) AT(IZ),DN(IZ)
      RADUS=SAMPKM*RADIUS
      DEPTH=SAMPKM*(ISAMP-1)-RADUS
      IF (DKM.LE.MINDKM.OR.DKM.GE.MAXDKM) GO TO 24
      WRITE (7,28) DISTA,DKM,TMIN,SEC,AT(IZ),RADUS,DEPTH,AMPL,MULTIP
      IF (IAAP.EQ.1) WRITE (7,31) APPVEL
   24 CONTINUE
      IVELOC=1
      IMERGE=1
      SLOPER=SLOPE
C     ****************PLOTTING INSTRUCTION****************
   25 IF (IPLOT.EQ.0) GO TO 26
      X(NPLT)=PJ(IZ)
      Y(NPLT)=PI(IZ)
      CALL PLINE (X(1),Y(1),NPLT,1,0,0,1)
      NPLT=0
   26 CONTINUE
C     ****************************************************
      IF (IMERGE.EQ.1) GO TO 22
      IF (IPP.EQ.1.AND.IMERGE.EQ.1) GO TO 22
      MULTIP=MULTIP-1
      IF (IZ.EQ.1) RETURN
      IZ=IZ-1
      JCRIT=0
      ANGLE2=CIJ(IZ)
      VELO3=VELOCY(IZ)
      VELO2=VELO3
      HORINC=SEGMEN*SIN(ANGLE2)
      VERINC=SEGMEN*COS(ANGLE2)
      IVELOC=1
      GO TO 5
C
   27 FORMAT (10F4.2)
   28 FORMAT ('',F6.2,F10.2,F7.2,F6.2,F8.2,2F9.2,F8.2,I6)
   29 FORMAT ('0INITIALANGLE,DEPTH,&DISTANCE'/'DELTA','KMMINSECTOTSECRAD
     1IUSDEPTHAMPL','MULTIPLES')
   30 FORMAT ('',3F6.2,3I3)
```

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31      FORMAT ('APPARENTVELOCITY=',F5.2)
        END
C
        SUBROUTINE BORD (PIR,PJR,ANGLE1,ISAMP,EDGE,ON,AT,DIFRAC,A,DISTA)
        RAMP=ISAMP-1.
        SAMP=ISAMP
        DO   J=1,30
        H=1.
        PINCI=EDGE*SIN(ANGLE1)
        PINCJ=EDGE*COS(ANGLE1)
        PRR=SAMP-PJR
        PII=PIR-1.
        RADI=SQRT(PRR*PRR+PII*PII)
        IF (RADI.LT.RAMP) GO TO 1
        PINCI=-PINCI
        PINCJ=-PINCJ
        H=-1.
1       PIR=PIR+PINCI
        PJR=PJR+PINCJ
        AT=AT+DIFRAC*H*EDGE/A
        ON=ON+DIFRAC*EDGE*H
        IF (RADI.GT.RAMP-EDGE.AND.RADI.LT.RAMP+EDGE) GO TO 3
2       CONTINUE
3       IF (ABS(PIR-1.).LT..0001) PIR=1.01
        ARG=(SAMP-PJR)/(PIR-1.)
        THETA=ATAN(ARG)
        DISTA=(1.5708-ABS(THETA))*57.2958
        IF (THETA.LT.0.) DISTA=(1.5708+ABS(THETA))*57.2956
        RETURN
        END
C
        SUBROUTINE RPOINT (ANGLE1,SLP,PIR,PJR,TSAMP,EDGE,VEL,ON,AT,SAMPKM,
       1VEL01,PIT,PIT2,ARG,SPEED,LIMINC,RADI,PRECI,EDGES,K1)
        NF=0.
        PII=PIR-1.
        PRR=TSAMP-PJR
        RADI=SQRT(PRR*PRR+PII*PII)
C       **********CHECK TO SEE IF RAY IS INWARD OR OUTWARD*************
        KVEL=K1
        IF (SPEED.LT.VEL) KVEL=K1-1
        RADTES=TSAMP-KVEL-PRECI
        IF (SPEED.LE.VEL) GO TO 1
        IF (RADTES.LT.RADI) GO TO 2
        GO TO 3
1       CONTINUE
        IF (RADI.LT.RADTES) GO TO 2
        GO TO 3
2       CONTINUE
        H=1.
        GO TO 4
3       H=-1.
4       CONTINUE
C       *********SET HORIZONTAL & VERTICAL INCREMENTS OF RAY**********
        PINCI=EDGE*SIN(ANGLE1)
        PINCJ=EDGE*COS(ANGLE1)
        DO 5 I=1,LIMINC
        PRR=TSAMP-PJR
        PII=PIR-1.
        RADI=SQRT(PRR*PRR+PII*PII)
C       ***********TEST FOR CLOSENESS TO RADIUS OF REFLECTION
```

```
      IF (ABS(RADTES-RADI).LT..6*EDGE) GO TO 6
      NE=NE+1.
C     *******INCREMENT & FIND TRAVELTIME & DISTANCE*************
      PIR=PIR+H*PINCI
      PJR=PJR+H*PINCJ
      AT=AT+H*SAMPKM*EDGE/VELO1
      DN=DN+H*SAMPKM*EDGE
    5 CONTINUE
    6 CONTINUE
      PIII=ABS(PII)
      IF (PIII.LT..0001) PIII=.0001
      IF (PII.LT.0.) PIII=-PIII
      ARG=PRR/PIII
      IF (ARG.GE.0.) GO TO 7
      SLP=ATAN(ARG)+PIT2
      GO TO 8
    7 CONTINUE
      SLP=ATAN(ARG)-PIT2
    8 CONTINUE
      EDGES=NE*EDGE*H
      RETURN
      END
```

```
C     PROGRAM FOR THREE-DIMENSIONAL MODEL OF EARTH SECTION
C     MODEL OF VELOCITY DISTRIBUTION IS SET UP AND RAYS ARE PROPAGATED
C     THROUGH THE MODEL
C     TRAVEL TIMES, DISTANCE IN KILOMETERS, AND
C     APPROXIMATE AMPLITUDES ARE COMPUTED
C                  SYMBOL TABLE
C     A       ARRAY OF SAMPLED VELOCITY VALUES
C     AEFF    PROBED VELOCITY VALUE
C     ADIF    MINIMUM ANGLE AT WHICH PRECISE INTERFACE LOCATION IS USED
C     AFFI    INCIDENT VELOCITY
C     ALPHA   ANGLE FROM ROTATED Z-AXIS
C     AMPL    AMPLITUDE
C     ANGLE2  EMERGING ANGLE IN X-Y PLANE
C     ANT     ARRAY FOR AMPLITUDES AFTER REFLECTIONS
C     AT      ARRAY FOR TRAVEL TIMES
C     B       ARRAY CONTAINING SAMPLED SLOPE VALUES
C     BASE    DISTANCE ALONG SURFACEIN KM
C     BD      ARRAY FOR DIPS
C     BDARG   COSINE OF MATHEMATICALLY DETERMINED INCLINATION
C     BDREF   SLOPE
C     BEGIN   INITIAL ANGLE OF RAY
C     BEGIND  INITIALIZATION FOR INCLINCATION
C     BMA     DIP
C     CAM     INTERMEDIATE VALUE FOR ROTATION
C     CIJ     ARRAY FOR VALUES OF ANGLES TO BE RETURNED TO AFTER REFLECTI
C     CKJ     ARRAY FOR SAVING REFRACTED INCLINATION AFTER REFLECTION
C     CLINE1  INCLINATION
C     CLINE2  EMERGING ANGLE FROM Z-AXIS
C     CLINEM  INCLINATION IN DEGREES
C     COMBEG  SUBROUTINE TO INCREMENT INITIAL ANGLE OF INCIDENCE
C     COMDEP  SUBROUTINE FOR INITIALIZING Z-VALUES
C     COMHOR  SUBROUTINE TO INCREMENT INITIAL HORIZONTAL POSITION
C     COMVER  SUBROUTINE TO INCREMENT INITIAL VERTICAL POSITION
C     CONST   CONSTANT FOR MATH FUNCTION
C     CPI     RE-ROTATED X-POSITION
C     CREF    COSINE OF INCLINATION
C     CX      PROJECTION ON ROTATED X AXIS
C     CY      PROJECTION ON Y-AAXIS IN ROTATED CORRDINATES
C     CZ      PROJECTION ON Z-AXIS IN ROTATED COORDINATES
C     DDD     INCREMENTATION IN Z-DIRECTION
C     DELD    INITIALIZATION INCREMENT FOR Z VALUE
C     DELN    INCREMENTATION ANGLE OF INCIDENT RAYS
C     DELV    INITIALIZATION INCREMENT IN Y
C     DIFRAC  DISTANCE IN KM BETWEEN SAMPLE POINTS
C     DISTA   DISTANCE IN KM FROM ORIGIN
C     DKM     DISTANCE IN KILOMETERS
C     DV      DISTANCE TO INTERFACE OR SURFACE
C     FACD    FACTOR WITH Z IN MATH FUNCTION
C     FACH    X FACTOR FOR MATH FUNCTION
C     FACV    FACTOR FOR Y-VALUE IN MATH FUNCTION
C     FFF     SINE OF EMERGING ANGLE(ABOVE 1.00 IF CRITICAL ANGLE)
C     G       RAY SEGMENT LENGTH
C     GGG     INCREMENTATION IN ROTATED Y-DIRECTION
C     GIN     INCREMENT TO INTERFACE OR SURFACE
C     GRAD    GRADIENT
C     HHH     X INCREMENT
C     HPERS   PLOTTING VARIABLE FOR PERSPECTIVE
C     IB      COUNTER FOR SUBROUTINE CALLS
C     IBD     COUNT FOR INITIALIZATION INCREMENTS IN Z
C     ICRIT   SWITCH TO INDICATE CRITICAL REFLECTION
```

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C       ICRYT   SWITCH FOR CRITICAL REFLECTIONS                                  A  61
C       ICYCLE  MAXIMUM NUMBER OF RAY INCREMENTATIONS                            A  62
C       IO      INTEGER VALUE OF TIO                                             A  63
C       IOTS    SWITCH USED IN PLOTTING                                          A  64
C       IEXACT  SWITCH TO INCLUDE PRECISE INTERFACE LOCATIONS                    A  65
C       IGIN    SWITCH FOR ROTATION LOOP                                         A  66
C       IH      COUNTER FOR HORIZONTAL INITIALIZATION INCREMENTS                 A  67
C       IM      INTEGER VALUE OF PROBED PI VALUE                                 A  68
C       IMATH   ARRAY FOR INDICATING MATH FUNCTION AFTER REFLECTION              A  69
C       IMI     SWITCH FOR RE-ROTATION LOOP                                      A  70
C       IPLANE  SWITCH FOR PLANE WAVE INITIALIZATIONS                            A  71
C       IPLOT   SWITCH TO INCLUDE CALCOMP PLOTTING                               A  72
C       IREFL   SWITCH FOR REFLECTIONS                                           A  73
C       IS      PROBED Y VALUE                                                   A  74
C       ISEC    TRAVEL TIME IN INTEGER NUMBER OF MINUTES                         A  75
C       ISURF   SWITCH TO INDICATE EMERGENCE TO SURFACE                          A  76
C       IWATH   ARRAY FOR RETAINING MATH FUNCTION AFTER REFLECTION               A  77
C       INUM    NUMBER OF TIMES THROUGH INNER LOOP                               A  78
C       IV      COUNTER FOR INITIALIZATIONS OF Z                                 A  79
C       IVFI    SWITCH FOR PLOTTING INSTRUCTION                                  A  80
C       IVU     SWITCH FOR REFLECTION                                            A  81
C       IVINI   SWITCH FOR ENTERING LOOP                                         A  82
C       IVX     SWITCH FOR REFLECTION                                            A  83
C       IVXX    SWITCH FOR REFLECTION                                            A  84
C       IVW     SWITCH FOR REACHING REFLECTION (CRITICAL OR NON-CRITICAL)        A  85
C       IWRITE  SWITCH TO INCLUDE DIAGNOSTIC WRITEOUTS                           A  86
C       IZ      INDEX FOR NUMBER OF RAY REFLECTION                               A  87
C       JMATH   SWITCH FOR MATH FUNCTION                                         A  88
C       JPORSH  LIMIT OF VERTICAL NUMBER OF VERTICAL SAMPLE POINTS               A  89
C       KMATH   SWITCH FOR USING MATH FUNCTION                                   A  90
C       KPORSH  LIMIT OF Y-VALUES                                                A  91
C       LMATH   SWITCH FOR MATHEMATICAL FUNCTION                                 A  92
C       MATH    SWITCH FOR MATH FUNCTION                                         A  93
C       MILZ    MAXIMUM LIMIT FOR MATH FUNCTION IN Z                             A  94
C       MILX    MAXIMUM X-VALUE FOR MATH FUNCTION                                A  95
C       MILY    MAXIMUM Y-VALUE FOR MATH FUNCTION                                A  96
C       MISX    MINIMUM LIMIT FOR MATH FUNCTION IN X                             A  97
C       MISY    MINIMUM Y-VALUE FOR MATH FUNCTION                                A  98
C       MISZ    MINIMUM Z VALUE FOR MATH FUNCTION                                A  99
C       NMATH   SWITCH FOR MATH FUNCTION                                         A 100
C       NMULT   NUMBER OF MULTIPLE REFLECTIONS ALLOWED                           A 101
C       NOA     NUMBER OF INITIALIZED ANGLES                                     A 102
C       NOD     NUMBER OF Z INITIALIZATION INCREMENTS                            A 103
C       NOH     INITIALIZATION INCREMENT FOR X VALUE                             A 104
C       NOIN    TOTAL NUMBER OF INITIAL RAYS                                     A 105
C       NOV     NUMBER OF Y-VALUE INITIALIZATION INCREMENTS                      A 106
C       NPLT    COUNTER FOR PLOTTING                                             A 107
C       NULINE  NUMBER OF CALCOMP LINES NECESSARY TO OUTLINE STRUCTURE           A 108
C       NUSYM1  SYMBOL FOR PLOTTING                                              A 109
C       NUSYM2  SYMBOL FOR PLOTTING                                              A 110
C       OBZ     PROJECTION ON Z AXIS                                             A 111
C       OD      ARRAY FOR DISTANCES RAYS HAVE TRAVELLED                          A 112
C       ORIGD   INITIAL DEPTH OF RAY SOURCE                                      A 113
C       ORIGH   INITIAL HORIZONTAL POSITION OF RAY                               A 114
C       ORIGV   INITIAL Y-VALUE                                                  A 115
C       PALFA   SIN ALPHA(FOR DETERMINING POSITION IN SPHERICAL COORDINATES      A 116
C       PALIK   DIP IN ANGLE FROM Z-AXIS                                         A 117
C       PERSLE  PLOTTING VARIABLE FOR PERSPECTIVE                                A 118
C       PI      ARRAY FOR X-VALUES                                               A 119
C       PII     CONSTANT: 3.14159                                                A 120
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C       PITT    CONSTANT: 1.5708                                              A 121
C       PJ      ARRAY FOR Y-VALUES                                            A 122
C       PK      ARRAY FOR Z-VALUES                                            A 123
C       PNM     INITIAL AMPLITUDE VALUE                                       A 124
C       REFALF  DUMMY VARIABLE OF ZERO FOR SLOPE IN SINDET                    A 125
C       REFVEL  REFERENCE VELOCITY                                            A 126
C       RID     PROJECTION OF PROBED Z POSITION IN ROTATED COORDINATES        A 127
C       RIGH    ORIGINAL ORIGN                                                A 128
C       RIM     PROBED X-VALUE IN ROTATED COORDINATES                         A 129
C       RIS     RE-ROTATED Y-POSITION                                         A 130
C       RRIM    INTERMEDIATE VALUE FOR RE-ROTATING COORDINATES                A 131
C       SHM     SINDE OF RIP                                                  A 132
C       SCA     PLOT SCALING FACTOR                                           A 133
C       SEC     TRAVEL TIME IN SECONDS ABOVE INTEGER MINUTES                  A 134
C       SLID    F.P. ID                                                       A 135
C       SLIM    FLOATING POINT IM                                             A 136
C       SLIS    FLOATING POINT VALUE OF IS                                    A 137
C       SREF    SIN MODEF                                                     A 138
C       THETA   ANGLE IN X-Y PLANE IN ROTATED COORDINATES                     A 139
C       TID     RE-ROTATED PROVED Z-VALUE                                     A 140
C       TIM     PROBED X POSITION                                             A 141
C       TIS     PROBED Y-VALUE                                                A 142
C       TMIN    TRAVEL TIME IN MINUTES                                        A 143
C       X       ARRAY CONTAINING HORIZONTAL POSITIONS FOR PLOTTING            A 144
C       XORTH1  ARRAY FOR PLOTTING                                            A 145
C       XORTH2  ARRAY FOR PLOTTING                                            A 146
C       XORTH3  ARRAY FOR PLOTTING                                            A 147
C       XPI     ROTATED VALUE OF PI(I2)                                       A 148
C       XPJ     ROTATED Y POSITION                                            A 149
C       XPK     ROTATED Z VALUE                                               A 150
C       XREF    X PROJECTION IN ROTATED COORDINATE SYSTEM                     A 151
C       XTID    PROBED VALUE IN ROTATED COORDINATES                           A 152
C       XTIM    PROBED X-VALUE AFTER ROTATION                                 A 153
C       XTIS    PROBED Y-VALUE IN ROTATED COORDINATES                         A 154
C       XX      INTERMEDIATE VALUE IN ROTATING COORDINATES                    A 155
C       XXPI    INTERMEDIATE VALUE FOR ROTATION                               A 156
C       XXR     INTERMEDIATE VALUE FOR RE-ROTATION                            A 157
C       XXTIM   INTERMEDIATE VALUE FOR ROTATING COORDINATES                   A 158
C       Y       ARRAY CONTAINING VERTICAL POSITIONS FOR PLOTTING              A 159
C       YORTH1  ARRAY FOR PLOTTING                                            A 160
C       YORTH2  ARRAY FOR PLOTTING                                            A 161
C       YORTH3  ARRAY FOR PLOTTING                                            A 162
C       YREF    ROTATED Y-VALUE                                               A 163
C       ZPJ     ROTATED Y-VALUE                                               A 164
C       ZPK     Z PROJECTION OF RAY SEGMENT IN RE-ROTATED COORDINATES         A 165
C       ZREF    ROTATED Z POSITION                                            A 166
C       ZTE     SAVED EMERGING POSITION FOR TESTING                           A 167
C       ZTEST   EMERGING POSITION FOR TESTING                                 A 168
C       VMATH1  COMPUTED VALUE OF MATH FUNCTION                               A 169
C       VMATH2  VELOCITY VALUE                                                A 170
C       VMATH3  VELOCITY VALUE                                                A 171
C       VPERS   PLOTTING VARIABLE FOR PERSPECTIVE                             A 172
C                                                                             A 173
        DIMENSION PI(10), PJ(10), CIJ(10), ON(10), ANT(10), AT(10), CKJ(10    A 174
       1), PK(10), IMATH(10)                                                  A 175
        DIMENSION A(30,30,30), B(30,30,30), AD(30,30,30), SLPRAT(30,30,30)    A 176
C             ***PLOTTING INSTRUCTIONS***                                     A 177
        DIMENSION X(200), Y(200), XORTH1(200), YORTH1(200), XORTH2(200), Y    A 178
       1ORTH2(200), XORTH3(200), YORTH3(200)                                  A 179
C             ***                      ***                                    A 180
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```
      DATA PII,PIII,ICRIT,ICRYT,INUM,ISURF,IR,IV,IH,IHD/3.1415927,1.5707
     1963,0,0,0,0,0,0,0,0/
      NAMELIST /NAM1/ NDIM,DELN,BEGIN,NMULT,ORIGV,ORIGH,G,IPORSH,JPORSH,
     1INFRAC,VELDIF,IWRITE,BEGIND,DELD,DEC,INAM1,NUR,NOV,NUH,NUD,DELV,DE
     2LH,ORIGD,KPORSH,ICYCLE,REEVEL,IEXACT,PNM,ADIF,X,Y,XORTH1,YORTH1,XO
     3RTH2,YORTH2,XORTH3,YORTH3,NOLINE,SCA,IPLANE,IR,IV,IH,PERSLE,VPERS,
     4CONST,FACH,FACV,FACD,NMATH,IPLOT,MISX,MILX,MISY,MILY,NPERS,NUSYM1,
     5NUSYM2,MIS2,MIL2/NAM2/A,B,ND,SLPRAT
      READ 16,NAM1
      PII=ORIGD
      IF (INAM1.EQ.1) GO TO 1
      READ 16,501 (((A(I,J,K),I=1,IPORSH),J=1,JPORSH),K=1,KPORSH)
      READ 16,501 (((B(I,J,K),I=1,IPORSH),J=1,JPORSH),K=1,KPORSH)
1     READ 16,NAM2
      WRITE 17,531
      WRITE 17,521
      CALL PLTDFS (10.,SCA,0.,SCA,4.,3.)
C         ***PLOTTING INSTRUCTIONS***
      IF (IPLOT.EQ.0) GO TO 2
      CALL PLINE (X(1),Y(1),NOLINE,1,0,0,1)
      CALL PLINE (XORTH1(1),YORTH1(1),5,1,0,0,1)
      CALL PLINE (XORTH2(1),YORTH2(1),5,1,0,0,1)
      CALL PLINE (XORTH3(1),YORTH3(1),5,1,0,0,1)
C         ***                        ***
2     CONTINUE
      DO 40 I=1,NDIM
      IF (IWRITE.GE.1) WRITE 17,551 IWRITE
      IF (BEGIN.GT.2.*PII) BEGIN=BEGIN-2.*PII
      REEALF=0.
      NPIT=0
      WRITE (7,561 BEGIN,BEGIND,ORIGH,ORIGV
      ANGLE2=BEGIN
      CLINE2=BEGIND
      IF (CLINE2.LT.0.) CLINE2=-CLINE2
      IF (CLINE2.GT.PII) CLINE2=2.*PII-CLINE2
      DO 3 I=1,10
      AT(I)=0.
3     DN(I)=0.
      IVV=0
      IVH=0
      IVX=3
      IMI=0
      I7=1
      IDTS=0
      MATH=NMATH
      IMATH(17)=MATH
      JMATH=MATH
      KMATH=MATH
      LMATH=MATH
      IVXX=3
      IVHH=0
      IGIN=0
      ANT(17)=1.
      PI(17)=ORIGH
      PJ(17)=ORIGV
      PK(17)=ORIGD
4     CONTINUE
      IREFL=0
      I=PI(17)+.5001
      J=PJ(17)+.5001
```

```
      K=PK(I7)+.500)
      IF (MATH.NE.1) GO TO 5
      AMA=PIT/2.-ATAN(FACH/FACV)
      HDARG=FACD/SQRT(FACD*FACD+FACV*FACV+FACH*FACH)
      HDREF=ARCOS(HDARG)
      GO TO 6
5     CONTINUE
      AMA=PIT/2.-HI(.,I,K)
      HDREF=HD(.,I,K)
6     CONTINUE
      CAM=COS(AMA)
      SAM=SIN(AMA)
      CREF=COS(HDREF)
      SREF=SIN(HDREF)
      XXPI=PI(I7)*CAM+PJ(I7)*SAM
      XPJ=PJ(I7)*CAM-PI(I7)*SAM
      XPI=XXPI*CREF-PK(I7)*SREF
      XPK=PK(I7)*CREF+XXPI*SREF
      CZ=COS(CLINE2)
      DRZ=ABS(SIN(CLINE2))
      CX=DRZ*SIN(ANGLE2)
      CY=DRZ*COS(ANGLE2)
      XX=CX*CAM+CY*SAM
      YREF=CY*CAM-CX*SAM
      XREF=XX*CREF-CZ*SREF
      ZREF=CZ*CREF+XX*SREF
      IF (ABS(YREF).LE..005) YREF=.005
      THETA=ATAN(XREF/YREF)
      IF (YREF.LT.0.) THETA=PIT+THETA
      IF (XREF.LT.0..AND.YREF.GE.0.) THETA=2.*PIT+THETA
      ALPHA1=ARCOS(ZREF)
      GRAD=G*SQRT(FACH*FACH+FACD*FACD+FACV*FACV)
7     IF (I7.LE.0) GO TO 48
      NPLT=NPLT+1
      INUM=INUM+1
      IF (INUM.GE.ICYCLE) CALL SYSTEM
      ALPHA=ALPHA1
      ANGLE1=ANGLE2
      CLINE1=CLINE2
C          ***PLOTTING INSTRUCTION***
      IF (IPLOT.EQ.0) GO TO 8
      IF (IDIS.EQ.0.OR.NPLT.GT.1) GO TO 9
      X(NPLT)=PJ(I7)-(PK(I7)-1.)*(VPERS+PJ(I7))/PERSLE
      Y(NPLT)=PI(I7)+(PK(I7)-1.)*(HPERS-PI(I7))/PERSLE
      XORTH1(1)=PJ(I7)+50.
      YORTH1(1)=PI(I7)+50.
      XORTH2(1)=31.-PK(I7)
      YORTH2(1)=PI(I7)+50.
      XORTH3(1)=PJ(I7)+50.
      YORTH3(1)=31.-(PK(I7))
      NPLT=NPLT+1
C        ***********              ***********
8     CONTINUE
9     DRLTO=G*ABS(SIN(ALPHA))
      HHH=DRLTO*SIN(THETA)
      GGG=DRLTO*COS(THETA)
      DDD=G*COS(ALPHA)
      IF (IREFL.EQ.1.OR.IVUU.EQ.1) GO TO 15
      IF (IVU.EQ.1) GO TO 24
      XPI=XPI+HHH
```

```
      XPJ=XPJ+GGG
      XPK=XPK+DDD
   10 CONTINUE
      XXPI=XPI*CREF+XPK*SREF
      PK(IZ)=-XPI*SREF+XPK*CREF
      PI(IZ)=XXPI*CRM-XPJ*SRM
      PJ(IZ)=XXPI*SRM+XPJ*CRM
      IF (IGIP.EQ.1) GO TO 41
      IF (MATH.NE.1) GO TO 11
      VMATH1=CONST+FACH*(PI(IZ)-1.)+FACV*(PJ(IZ)-1.)+FACD*(PK(IZ)-1.)
      GO TO 12
   11 CONTINUE
      VMATH1=A(I,J,K)
   12 CONTINUE
      AT(IZ)=AT(IZ)+DIFRAC*G/VMATH1
      ON(IZ)=ON(IZ)+DIFRAC*G
      IF (IPLOT.EQ.0) GO TO 13
C        ***PLOTTING INSTRUCTION***
      IDIS=1
      IF (IVEL.EQ.1.AND.NPLT.GE.3) NPLT=NPLT-1
      X(NPLT)=PJ(IZ)-(PK(IZ)-1.)*(VPERS+PJ(IZ))/PERSLE
      Y(NPLT)=PI(IZ)+(PK(IZ)-1.)*(HPERS-PI(IZ))/PERSLE
      XORTH1(NPLT)=PJ(IZ)+50.
      YORTH1(NPLT)=PI(IZ)+50.
      XORTH2(NPLT)=31.-PK(IZ)
      YORTH2(NPLT)=PI(IZ)+50.
      XORTH3(NPLT)=PJ(IZ)+50.
      YORTH3(NPLT)=31.-PK(IZ)
C        ***                     ***
   13 CONTINUE
      IF (IWRITE.GE.2) WRITE (7,58) ALPHA,THETA,ICRYT,IZ,MATH
      IF (PK(IZ).GE.1.) GO TO 19
   14 ISURF=1
   15 CONTINUE
      XXR=HHH*CREF+DDD*SREF
      ZPK=-HHH*SREF+DDD*CREF
      ZPI=XXR*CRM-GGG*SRM
      ZPJ=XXR*SRM+GGG*CRM
      IF (ABS(ZPJ).LE..005) ZPJ=.005
      ANGLE1=ATAN(ZPI/ZPJ)
      IF (ZPJ.LT.0.) ANGLE1=PIT+ANGLE1
      IF (ZPI.LT.0..AND.ZPJ.GE.0.) ANGLE1=2.*PIT+ANGLE1
      CLINE2=ARCOS(ZPK)
      IF (IVU.EQ.1) GO TO 27
      IF (IVUU.EQ.1) GO TO 44
      IF (IM1.EQ.1) GO TO 25
      IF (IREFL.EQ.1) GO TO 4
      IF (MATH.NE.1) GO TO 16
      VMATH3=CONST+FACH*(PI(IZ)-1.)+FACV*(PJ(IZ)-1.)
      GO TO 17
   16 CONTINUE
      VMATH3=A(I,J,K)
   17 CONTINUE
      DV=1.-PK(IZ)
      IF (IEXACT.EQ.1) CALL GINMAD (PI(IZ),PJ(IZ),PK(IZ),DV,AT(IZ),ON(IZ
     1),DIFRAC,VMATH3,CLINE2,ANGLE1,ADIF)
      ISURF=0
      BASE=SQRT(PI(IZ)*PI(IZ)+PJ(IZ)*PJ(IZ))*DIFRAC
      DISTA=SQRT((PI(IZ)-ORIGH)*(PI(IZ)-ORIGH)+(PJ(IZ)-ORIGV)*(PJ(IZ)-OR
     1IGV))*DIFRAC
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      DKM=BASE-1.732*DIFRAC
      AMPL=(DKM*ANT(IZ)/DN(IZ))*EXP(DEC*DN(IZ))
      TMIN=AT(IZ)/60.
      ISEC=TMIN
      SEC=60.*(TMIN-ISEC)
      WRITE (7,56) DISTA,DKM,TMIN,SEC,AT(IZ),AMPL
      IF (IPLOT.EQ.0) GO TO 18
C         ***PLOTTING INSTRUCTIONS***
      X(NPLT)=PJ(IZ)-(PK(IZ)-1.)*(VPERS+PJ(IZ))/PERSLE
      Y(NPLT)=PI(IZ)+(PK(IZ)-1.)*(HPERS-PI(IZ))/PERSLE
      XORTH1(NPLT)=PJ(IZ)+50.
      YORTH1(NPLT)=PI(IZ)+50.
      XORTH2(NPLT)=31.-PK(IZ)
      YORTH2(NPLT)=PI(IZ)+50.
      XORTH3(NPLT)=PJ(IZ)+50.
      YORTH3(NPLT)=31.-PK(IZ)
      CLINEM=57.2958*CLINE1
      CALL PLINE (X(1),Y(1),NPLT,1,0,0,1)
      CALL PSYMB (X(NPLT),Y(NPLT),.1,NUSYM2,CLINEM,-1,1)
      CALL PLINE (XORTH1(1),YORTH1(1),NPLT,1,0,0,1)
      CALL PSYMB (XORTH1(NPLT),YORTH1(NPLT),.1,NUSYM2,CLINEM,-1,1)
      CALL PLINE (XORTH2(1),YORTH2(1),NPLT,1,0,0,1)
      CALL PLINE (XORTH3(1),YORTH3(1),NPLT,1,0,0,1)
      NPLT=0
C         ***                         ***
18    CONTINUE
      GO TO 22
19    IF (PI(IZ).GT..5.AND.PJ(IZ).GT..5.AND.PJ(IZ).LT.JPORSH-.5.AND.PI(I
     1Z).LT.IPORSH-.5.AND.PK(IZ).LT.KPORSH-.5) GO TO 23
20    IF (IPLOT.EQ.0) GO TO 21
      CALL PLINE (X(1),Y(1),NPLT,1,0,0,1)
      CALL PLINE (XORTH1(1),YORTH1(1),NPLT,1,0,0,1)
      CALL PLINE (XORTH2(1),YORTH2(1),NPLT,1,0,0,1)
      CALL PLINE (XORTH3(1),YORTH3(1),NPLT,1,0,0,1)
      NPLT=0
21    CONTINUE
22    IF (IZ.EQ.1) GO TO 40
      IVW=0
      ANT(IZ-1)=(1-ANT(IZ))*ANT(IZ-1)
      IZ=IZ-1
      ANGLE2=CLJ(IZ)
      CLINE2=CKJ(IZ)
      MATH=IMATH(IZ)
      JMATH=MATH
      IF (MATH.EQ.1) IVX=3
      IF (MATH.EQ.0) IVXX=3
      GO TO 4
23    I=PI(IZ)+.5
      J=PJ(IZ)+.5
      K=PK(IZ)+.5
24    CONTINUE
      XTIM=XPI+HHH
      XTIS=XPJ+GGG
      XTID=XPK+DDD
C
      XXTIM=XTIM*CBEE+XTID*SBEE
      TID=-XTIM*SBEE+XTID*CBEE
      TIM=XXTIM*CAM-XTIS*SAM
      TIS=XXTIM*SAM+XTIS*CAM
      IM=TIM+.5
```

```
      IS=TIS+.5
      ID=TID+.5
      DV=SLPRAT(I,J,K)/100.
      IF (DV.LT..01) DV=.5
      IF (ALPHA.LT.PITT) SLID=ID-DV
      IF (ALPHA.GE.PITT) SLID=ID+1.-DV
      SLIM=IM
      SLIS=IS
      RRIM=SLIM*CBM+SLIS*SBM
      RIS=SLIS*CBM-SLIM*SBM
      RIM=RRIM*CREF-SLID*SREF
      RID=SLID*CREF+RRIM*SREF
      DV=RID-XPK
      IF (IVU.EQ.1) GO TO 29
C     TEST TO SEE IF RAY END IS IN SAMPLED REGION.  SET SWITCHES
C     TO CORRESPOND TO THE REGION.  IF NECESSARY, RECOMPUTE
C     ROTATION OF COORDINATES AND RESULTING DIRECTIONS...
      IF (IM.GE.MISX.AND.IM.LT.MILX.AND.IS.GE.MISY.AND.IS.LE.MILY.AND.ID
     1.GE.MISZ.AND.ID.LE.MILZ) GO TO 26
      IVX=IVX+1
      IVXX=0
      IF (MATH.EQ.1.AND.IVX.GE.3) GO TO 29
      MATH=1
      IMATH(IZ)=1.
      IF (IVX.EQ.2) KMATH=1
      LMATH=1
      IF (IVX.EQ.1) GO TO 29
      IMI=1
      GO TO 15
   25 CONTINUE
      IMI=0
      IVXX=0
      CBM=COS(BMA)
      SBM=SIN(BMA)
      CREF=COS(BDREF)
      SREF=SIN(BDREF)
      GO TO 28
   26 CONTINUE
      IVXX=IVXX+1
      IF (IVXX.EQ.2) LMATH=0
      KMATH=0
      IVX=0
      IF (MATH.EQ.0) GO TO 29
      MATH=0
      IMATH(IZ)=0
      IVU=1
      GO TO 15
   27 CONTINUE
      PRBJK=PIT-B(I,J,K)
      CBM=COS(PRBJK)
      SBM=SIN(PRBJK)
      CREF=COS(BD(I,J,K))
      SREF=SIN(BD(I,J,K))
   28 CONTINUE
      XXPI=PI(IZ)*CBM+PJ(IZ)*SBM
      XPJ=PJ(IZ)*CBM-PI(IZ)*SBM
      XPI=XXPI*CREF-PK(IZ)*SREF
      XPK=PK(IZ)*CREF+XXPI*SREF
      CZ=COS(CLINE2)
      OBZ=ABS(SIN(CLINE2))
```

```
      CX=DHZ*SIN(ANGLE1)
      CY=DHZ*COS(ANGLE1)
      XX=CX*CHM+CY*SHM
      YREF=CY*CHM-CX*SHM
      XREF=XX*CREF-CZ*SREF
      ZREF=CZ*CREF+XX*SREF
      IF (ABS(YREF).LE..005) YREF=.005
      THETA=ATAN(XREF/YREF)
      IF (YREF.LT.0.) THETA=PIT+THETA
      IF (XREF.LT.0..AND.YREF.GE.0.) THETA=2.*PIT+THETA
      ALPHA1=ARCOS(ZREF)
      ALPHA=ALPHA1
      IF (IVU.EQ.1) GO TO 9
29    CONTINUE
      IVU=0
      IF (MATH.NE.1) GO TO 30
      VMATH2=CONST+FACH*TIM+FACV*TIS+FACD*TID
      GO TO 31
30    CONTINUE
      VMATH2=A(IM,IS,ID)
31    CONTINUE
      IF (ID.LT.1) GO TO 14
      IF (IM.GT.IPORSH.OR.IS.GT.JPORSH.OR.IM.LT.1.OR.IS.LT.1.OR.ID.GT.KP
     1ORSH) GO TO 20
      IF (IWRITE.EQ.0) GO TO 32
      WRITE (7,51) PJ(IZ),PI(IZ),PK(IZ),VMATH1
C
32    IF (ABS(VMATH2-VMATH1).GT..0001) GO TO 33
      IVEL=1
      IVW=0
      GO TO 7
33    IVEL=0
      IF (IVW.NE.1) GO TO 34
      IVW=0
      GO TO 7
34    CONTINUE
      AFFI=VMATH1
      GRAD=G*SQRT(FACH*FACH+FACD*FACD+FACV*FACV)
      ALPHA2=ALPHA
C     LOOP TO DETERMINE EMERGING VELOCITY AND ANGLE****
      DO 38 III=1,10
      IF (MATH.EQ.1.AND.JMATH.EQ.MATH) GO TO 35
      ABFE=VMATH1+(VMATH2-VMATH1)*ABS(COS(ALPHA1))
      GO TO 36
35    CONTINUE
      ABFE=VMATH1+GRAD*COS(ALPHA1)
36    CONTINUE
      CALL SINDET (ALPHA,ALPHA1,AFFI,ABFE,REFALF,ICRYT,FFF)
      IF (ALPHA1.GT.PIT.AND.ALPHA1.LT.2.*PIT) THETA=THETA+PIT
      IF (ALPHA1.GT.PIT) ALPHA1=2.*PIT-ALPHA1
      IF (IWRITE.GE.3) WRITE (7,54) AFFI,ABFE,ALPHA1,THETA
      ZTEST=PK(IZ)+G*COS(ALPHA1)
      IF (III.EQ.1) GO TO 37
      IF (ABS(ZTEST-ZTE).LE..01) GO TO 39
37    CONTINUE
      ZTE=ZTEST
38    CONTINUE
C     END OF EMERGING ANGLE AND VELOCITY DETERMINATION LOOP****
39    CONTINUE
      JMATH=MATH
```

```
      ALPHA=ALPHA1
      IF (ABS(VMATH1-ABEF).GE.VELDIF.AND.IZ.LE.NMULT.AND.IVW.LT.1) GO TO
     1 40
      IF (ICRYT.EQ.1) GO TO 43
      IVW=0
      IBEFI=0
      GO TO 7
   40 CONTINUE
      IF (IWRITE.GE.4) WRITE (7,57) XPI,XPJ,XPK,MATH,KMATH,LMATH
      IF (IEXACT.EQ.1) CALL GINMAD (XPI,XPJ,XPK,DV,AT(IZ),ON(IZ),DIFRAC,
     1AEFI,ALPHA2,THETA,ADIF)
      IF (IPLOT.EQ.0) GO TO 42
      IGIN=1
      GO TO 10
   41 CONTINUE
      IGIN=0
C           ***PLOTTING INSTRUCTION***
      X(NPLT)=PJ(IZ)-(PK(IZ)-1.)*(VPERS+PJ(IZ))/PERSLF
      Y(NPLT)=PI(IZ)+(PK(IZ)-1.)*(HPERS-PI(IZ))/PERSLF
      XORTH1(NPLT)=PJ(IZ)+50.
      YORTH1(NPLT)=PI(IZ)+50.
      XORTH2(NPLT)=31.-PK(IZ)
      YORTH2(NPLT)=PI(IZ)+50.
      XORTH3(NPLT)=PJ(IZ)+50.
      YORTH3(NPLT)=31.-PK(IZ)
      CALL PLINE (X(1),Y(1),NPLT,1,0,0,1)
      CALL PLINE (XORTH1(1),YORTH1(1),NPLT,1,0,0,1)
      CALL PLINE (XORTH2(1),YORTH2(1),NPLT,1,0,0,1)
      CALL PLINE (XORTH3(1),YORTH3(1),NPLT,1,0,0,1)
      NPLT=0
C           ***                          ***
   42 CONTINUE
      IF (IWRITE.GE.4) WRITE (7,56) GIN,THETA,ALPHA2,XPI,XPJ,XPK
   43 IF (ICRYT+IVW.GE.2) GO TO 7
      IF (ICRYT.EQ.1) GO TO 45
      IVW=1
      IZ=IZ+1
      ON(IZ)=ON(IZ-1)
      AT(IZ)=AT(IZ-1)
      PI(IZ)=PI(IZ-1)
      PJ(IZ)=PJ(IZ-1)
      PK(IZ)=PK(IZ-1)
      ANT(IZ)=ANT(IZ-1)*(VMATH2-VMATH1)/(VMATH2+VMATH1)
      IVUU=1
      GO TO 9
   44 CONTINUE
      IVUU=0
      CIJ(IZ-1)=ANGLE1
      CKJ(IZ-1)=CLINE2
      IMATH(IZ-1)=MATH
   45 IF (IZ.LE.NMULT) GO TO 46
      GO TO 22
   46 CONTINUE
      IF (ICRYT.EQ.1.OR.ABS(ABEF-AEFI).GE.VELDIF) ALPHA1=PIT-ALPHA2
      DO 47 I=1,2
      IF (ALPHA1.LT.0.) ALPHA1=-ALPHA1
      IF (ALPHA1.GT.PIT) ALPHA1=2.*PIT-ALPHA1
      ALPHA=ALPHA1
   47 CONTINUE
      IMATH(IZ)=MATH
```

```
      IF (MATH.EQ.1.AND.KMATH.EQ.0) MATH=0
      IF (IMATH(IZ).EQ.0.AND.IMATH.EQ.1) MATH=1
      IMATH(IZ)=MATH
      IF (ABS(AFFI-ABFF).GE.VELDIF) IVW=1
      IF (MATH.EQ.0) GO TO 7
      IREFL=1
      GO TO 9
48    CONTINUE
      CALL COMBEG (BEGIN,DELN,NOB,IB)
      CALL COMDEP (BEGIND,DELD,NOD,IBD)
      CALL COMVER (ORIGV,DELV,NOV,IV)
      CALL COMHOR (ORIGH,DELH,NOH,IH)
49    CONTINUE
C
C           ***PLOTTING INSTRUCTION***
      CALL PLTEND
C
50    FORMAT (10F4.2)
51    FORMAT ('',5F8.2)
52    FORMAT ('DISTANCEKILOMETERSTT(MIN)TT(SEC)','TOTAL(SEC)AMPLITUDE')
53    FORMAT ('0INITIALANGLE,INITIALDISTANCE,INITIALDEPTH')
54    FORMAT ('',4F8.2)
55    FORMAT ('0CONDITIONALWRITEOUTSASFOLLOWS:'/'IWRITE=1:PI,PJ,PK,VMATH
     1)'/'IWRITE=2:ADD:ALPHA,THETA,ICRYT,IZ,MATH'/'IWRITE=3:ADD:AFFI,ABF
     2F,ALPHA1,THETA'/'IWRITE=4:ADDTWOLINES:'/'1,XPI,XPJ,XPK','MATH,KMAT
     3H,IMATH'/'AND2,GIN,THETA,ALPHA2,XPI,XPJ'/'IWRITE=',I2)
56    FORMAT ('',6F10.2)
57    FORMAT ('',3F10.2,3I2)
58    FORMAT ('',2F5.2,3I2)
      END
      SUBROUTINE GINMAD (PI,PJ,PK,DV,AT,ON,D,A,CLINE2,ANGLE1,ADIF)
      IF (ABS(CLINE2-1.57).LT.ADIF) RETURN
      CLICOS=COS(CLINE2)
      IF (ABS(CLINE2-PIE2).LT..01) CLICOS=.01
      GIN=DV/CLICOS
      OBLIO=GIN*SIN(CLINE2)
      PI=PI+OBLIO*SIN(ANGLE1)
      PJ=PJ+OBLIO*COS(ANGLE1)
      PK=PK+GIN*COS(CLINE2)
      IF (A.LT.1.) A=6.
      AT=AT+D*GIN/A
      ON=ON+D*GIN
      RETURN
      END
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13. ABSTRACT

Simulation of seismic rays for a spherical earth and a flat earth has been achieved in highly complex models. Travel times and approximate amplitudes of seismic waves can be found for both two- and three-dimensional models of portions of the earth. In seismology and other disciplines ray construction customarily has been applied to simplified geometries. It has been necessary to assume that the seismic wave velocity distribution of the earth was relatively uniform and symmetric.

Recently, however, the earth has been found to be more complex and non-uniform than formerly assumed. A need has thus arisen in seismology to test highly heterogeneous models of seismic velocity distribution. At the same time the development of the modern digital computer has provided a means of performing the necessary ray constructions and numerical calculations.

The problem of complicated seismic velocity distributions was therefore investigated in terms of the most appropriate use of the digital computer. For this investigation a velocity field was set up, and the propagation computations made for short segments of rays within this field. Total travel times are found by adding the travel times of connected ray segments. Essentially, the nature of propagation was duplicated on the computer, in that, at the location of each segment along the path of propagation, the initial condition and effect of the surroundings determine the succeeding direction of the following segment.

Both visual and numerical results have shown that this simulation method can be usefully applied to investigation of seismic velocity distributions of portions of the earth of any size or complexity.

14 KEY WORDS

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